Test 2 (A) **Introduction to Linear Algebra** **June 15, 2018**

MATH 313

Name: ______________________ **Signature:** ______________________

SHOW ALL YOUR WORK!

1. [16 points] Let A and B be 3×3 matrices, with $\det A = 3$ and $\det B = -4$. Find the following:
   (a) $\det[A^{-1}BA]$.
   (b) $\det[3B^TA]$.
   (c) $\det[A^2B]$.
   (d) $\det[(BA^{-1})^{-1}]$

2. [15 points] Solve **THREE** from the following four questions. Explain your answers!

   (a) Determine if the set H of all polynomials of the form $\mathbf{p}(t) = 2 + t^2$ is a subspace of $\mathbb{P}_5$.

   (b) Determine if the set H of all polynomials in $\mathbb{P}_n$ such that $\mathbf{p}(0) = 0$, is a subspace of $\mathbb{P}_n$..

   (c) Let H be the set of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, where a and b are real numbers. Determine if H is a subspace of $M_{2\times 2}$.

   (d) Let

$$H = \left\{ \begin{bmatrix} a+b \\ b \\ 2a+b \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

   Determine if H is a subspace of $\mathbb{R}^3$.

3. [15 points] Let $M_{2\times 2}$ be the vector space of all 2×2 matrices and define $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A - A^T$, where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

(a) Show that T is a linear transformation.

(b) Describe the kernel of T.

(c) Find a basis $\mathcal{B}$ of $M_{2\times 2}$, then find $[A]_{\mathcal{B}}$.

4. [15 points] Find bases for Null A and Col A, where

$$A = \begin{bmatrix} 2 & -4 & 2 & 4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$$

5. [15 points] Use **coordinate vectors** to test the linear independence of the set of polynomials. Explain your work!

$$1 + 2t^3,\ 2 + t - 3t^2,\ -t + 2t^2 - t^3.$$

6. [16 points] With no explicit calculations, find one eigenvalue for each of the following matrices, explain your reasoning

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 5 & 5 \\ 2 & 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & 2 \\ 0 & 5 & 1 \\ 0 & -1 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & -1 \\ 3 & -1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} -2 & -1 & 5 \\ 0 & -3 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

7. [16 points] In the following, A is an $n \times n$ matrix. Mark each statement as **True** or **False**.

- (   ) If A is invertible, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$ in $\mathbb{R}^n$.
- (   ) If A is invertible, then elementary row operations that reduce A to the identity I_n also reduce A^{-1} to I_n.
- (   ) If A^T is not invertible then A is not invertible.
- (   ) If the columns of A span $\mathbb{R}^n$ then they are linearly independent.
- (   ) If the columns of A are linearly independent then they span $\mathbb{R}^n$.
- (   ) If A is invertible then the columns of A^{-1} are linearly independent.
- (   ) If A is invertible then the inverse of $2A^T A^{-1}$ is $2A(A^{-1})^T$.
- (   ) If $\det A$ is zero, then two rows or two columns are the same, or a row or a column is zero.
- (   ) $\det(rA) = r \det A$ where r is in $\mathbb{R}$.
- (   ) The range of a linear transformation is a vector space.
- (   ) Let B be an $n \times m$ matrix. The set of all solutions of the homogeneous system $B\mathbf{x} = \mathbf{0}$ is a subspace of $\mathbb{R}^m$.
- (   ) If A is invertible then the number 0 is not an eigenvalue of A^T.
- (   ) If $H = \text{span}\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ then $\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ is a basis for H.
- (   ) A basis is a spanning set that is as large as possible.
- (   ) If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.
- (   ) The number of free variable in the equation $A\mathbf{x} = \mathbf{0}$ equals the dimension of $\text{Null}\, A$.