MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 2
SUMMER 2018

Due on: Friday 05-25-2018.

MATH 413: Solve questions 1 to 4.
MATH 513: Solve questions 1 to 5.

Vector spaces:
In the following, $V$ is a vector space over the field $F$.

(1) Let $p(z)$ be a polynomial with real coefficient, and let $\alpha \in \mathbb{C}$. Prove that $p(\alpha) = 0$ if and only if $p(\overline{\alpha}) = 0$.

(2) Prove that $-(-v) = v$ for every $v \in V$.

(3) For $v, w \in V$, explain why there exists a unique $x \in V$ such that $v + \alpha x = w$, for any nonzero $\alpha \in F$.

(4) The empty set is not a vector space, explain!

(5) Show that the additive inverse in the definition of the vector space can be replaced with the condition that

$$0v = 0, \text{ for all } v \in V.$$

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