MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 4 - SUMMER 2018

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Due on: Wednesday 06-6-2018.

MATH 413: Solve questions 1 to 4.
MATH 513: Solve questions 1 to 5.

Vector spaces:
In the following, $V$ is a finite dimensional vector space over the field $F$.

(1) Prove or disprove: the list of vectors $(\sin^2(x), \cos(2x), \alpha)$ is linearly independent in $C(\mathbb{R})$.

(2) Find the dimension of the following subspace of $\mathbb{R}^3$

$$U = \{(x_1, x_2, x_3, x_4) \mid x_4 = x_1 + x_2\}.$$ 

(3) Let $\dim(V) = n$ for some $n \in \mathbb{Z}_+$. Prove that there are $n$ one-dimensional subspaces $U_1, U_2, \ldots, U_n$ of $V$ such that

$$V = U_1 \oplus U_2 \oplus \ldots \oplus U_n.$$ 

(4) Let $U = \{p \in \mathbb{F}_4[z] : p(6) = 0\}$.
   (a) Find a basis of $U$.
   (b) Extend the basis in part (a) to a basis of $\mathbb{F}_4[z]$.
   (c) Find a subspace $W$ of $\mathbb{F}_4[z]$ such that $\mathbb{F}_4[z] = U \oplus W$.

(5) Let $\mathbb{F}_m[z]$ denote the vector space of all polynomials with degree less than or equal to $n \in \mathbb{Z}_+$ and having coefficient over $\mathbb{F}$, and suppose that $p_0, p_1, \ldots, p_m \in \mathbb{F}_m[z]$ satisfy $p_j(1) = 0$. Prove that $(p_0, p_1, \ldots, p_m)$ is a linearly dependent list of vectors in $\mathbb{F}_m[z]$. 

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