MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 1
DR. ABDUL-RAHMAN
SUMMER 2020

• Submit questions 1 and 4 for grading (Upload to D2L).
• Due on: Monday 05/25/2019 before class.
• You will have a quiz from the following questions on Monday 05/25/2020.

(1) Write the following system of linear equations as an equation for a single function \( f : \mathbb{R}^n \rightarrow \mathbb{R}^m \) for appropriate choices of \( m, n \in \mathbb{Z}_+ \),
\[
\begin{align*}
x + 2y - 3z &= 4 \\
x + 3y + z &= 11 \\
2x + 5y - 4z &= 0 \\
x + y + z &= 22
\end{align*}
\]

(2) Solve the following equations for \( z \) a complex number:
(a) \( z^3 - 4i = 0 \).
(b) \( z^6 + 8 = 0 \).

(3) Find \( r > 0 \) and \( \theta \in [0, 2\pi) \) such that \( (1 - i)/\sqrt{2} = re^{i\theta} \).

(4) Show that for any \( z \in \mathbb{C} \)
(a) \( z^z - 4i = 0 \).
(b) \( z^6 + 8 = 0 \).

(5) Let \( z, w \in \mathbb{C} \). Prove the parallelogram law
\[
|z - w|^2 + |z + w|^2 = 2(|z|^2 + |w|^2).
\]

(6) Read Section 2.3.4 then prove Theorem 2.3.5: parts (1) and (2).

(7) Compute the real and the imaginary parts of \( e^{e^x} \) for \( x \in \mathbb{C} \).

(8) Given any complex number \( \alpha \in \mathbb{C} \), show that the coefficient of the polynomial
\[
(z - \alpha)(z - \overline{\alpha})
\]
are real numbers.

(9) Let \( z, w \in \mathbb{C} \) with \( \overline{z}w \neq 1 \) such that either \( |z| = 1 \) or \( |w| = 1 \). Prove that
\[
\left| \frac{z - w}{1 - \overline{z}w} \right| = 1
\]

(10) Given a polynomial \( p(z) = a_nz^n + \ldots + a_1z + a_0 \) with complex coefficients, define the \textbf{conjugate} of \( p(z) \) to be the new polynomial
\[
\overline{p}(z) = \overline{a_n}z^n + \ldots + \overline{a_1}z + \overline{a_0}.
\]
(a) Prove that \( \overline{p}(\overline{z}) = \overline{p(z)} \).
(b) Prove that \( p(z) \) has real coefficient if and only if \( \overline{p}(z) = p(z) \).
(c) Given polynomials \( p(z), q(z), \) and \( r(z) \) such that \( p(z) = q(z)r(z) \), prove that \( \overline{p}(z) = \overline{q(z)}r(z) \).