(1) Show that the vectors \( v_1 = (1, 1, 1), v_2 = (1, 2, 3), \) and \( v_3 = (2, -1, 1) \) are linearly independent in \( \mathbb{R}^3 \). Write \( v = (1, -2, 5) \) as a linear combination of \( v_1, v_2, \) and \( v_3 \).

(2) Define \( U = \text{span}(u_1, u_2, \ldots, u_n) \). Suppose \( v \in U \), prove 
\[
U = \text{span}(v, u_1, u_2, \ldots, u_n).
\]

(3) Suppose that \( v_1, v_2, \ldots, v_n \) is a linearly independent set of vectors in \( V \). Given any \( w \in V \) such that the set of vectors 
\[
v_1 + w, v_2 + w, \ldots, v_n + w
\]

is linearly dependent, prove that \( w \in \text{span}(v_1, v_2, \ldots, v_n) \).

(4) Suppose \( v_1, v_2, v_3, v_4 \) is a basis of \( V \). Prove that 
\[
v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4
\]

is also a basis of \( V \).

(5) Find the dimension of the following subspace of \( \mathbb{R}^4 \)
\[
U = \{(x_1, x_2, x_3, x_4) | x_4 = x_1 + x_2 \}.
\]

(6) Let \( \dim(V) = n \) for some \( n \in \mathbb{Z}_+ \). Prove that there are \( n \) one-dimensional subspaces \( U_1, U_2, \ldots, U_n \) of \( V \) such that 
\[
V = U_1 \oplus U_2 \oplus \ldots \oplus U_n.
\]

(7) Let \( U = \{p \in \mathcal{P}_4[\mathbb{F}] : p(6) = 0\} \).
   (a) Find a basis of \( U \).
   (b) Extend the basis in part (a) to a basis of \( \mathcal{P}_4[\mathbb{F}] \).
   (c) Find a subspace \( W \) of \( \mathcal{P}_4[\mathbb{F}] \) such that \( \mathcal{P}_4[\mathbb{F}] = U \oplus W \).

(8) Let \( U \) and \( V \) be four-dimensional subspace of \( \mathbb{R}^7 \). Prove that \( U \cap V \neq \{0\} \).

(9) Suppose \( p_0, p_1, \ldots, p_m \in \mathcal{P}[\mathbb{F}] \) are such that each \( p_j \) has degree \( j \). Prove that \( p_0, p_1, \ldots, p_m \) is a basis of \( \mathcal{P}_m[\mathbb{F}] \).

(10) Suppose that \( p_0, p_1, \ldots, p_m \in \mathcal{P}_m[\mathbb{F}] \) satisfy \( p_j(1) = 0 \). Prove that \( (p_0, p_1, \ldots, p_m) \) is a linearly dependent list of vectors in \( \mathcal{P}_m[\mathbb{F}] \).