(1) [20 points] Write the following system of linear equations as an equation for a single function $f : \mathbb{R}^n \to \mathbb{R}^m$ for appropriate choices of $m, n \in \mathbb{Z}_+$,

\[
\begin{align*}
  x + 2y - 3z + w &= 11 \\
  x + 3y + z - w &= -1 \\
  y + w &= 0
\end{align*}
\]

(2) [20 points] Solve the following equation for $z \in \mathbb{C}$,

$z^3 - i = 1$.

(3) [20 points] Let $U_1$ and $U_2$ be two subspaces of a vector space $V(\cdot, \cdot)$. In each of the following, prove or give a counter example:

(a) $U_1 \cap U_1$ is a subspace of $V$.
(b) $U_1 \cup U_2$ is a subspace of $V$.
(c) $U_1 - U_2$ is a subspace of $V$, where $U_1 - U_2 := \{u_1 + (-u_2); \ u_1 \in U_1, \ u_2 \in U_2\}$.

(4) [20 points] Let

$U := \{(x, x, 0) \in \mathbb{R}^3; \ x \in \mathbb{R}\}$

(a) Show that $U$ is a subspace of $\mathbb{R}^3$.
(b) Find a subspace $W$ of $\mathbb{R}^3$ such that $\mathbb{R}^3 = U \oplus W$.

(5) [20 points] Consider the vector space $\mathbb{R}^{[0,1]}$ of functions $f : [0, 1] \to \mathbb{R}$. Define

$S = \left\{ f \in \mathbb{R}^{[0,1]}; \ f \text{ is continuous on } [0, 1] \text{ and } \int_0^1 f(x) \, dx = 0 \right\}$

Show that $S$ is a subspace of $\mathbb{R}^{[0,1]}$. \}