TEST II

12 questions, 10 points each. SHOW ALL YOUR WORK!

Question 1

Find the area of the region bounded by the parabola $y = -2x - x^2$ and the line $y = 2x$. 
Question 2

Find the volume of the solid obtained by rotating about the $x$-axis the region bounded by $y = \sqrt{x}$ and the $x$-axis and the line $x = 1$.

Question 3

Decide whether the following series is convergent or divergent, if it is convergent find its sum

\[
\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \ldots
\]
Question 4

Determine whether the following sequence is convergent or divergent, if it is convergent find its limit.

\[ a_n = \ln(n + 1) - \ln(2n - 5). \]

Question 5

Determine whether the series is absolutely convergent, conditionally convergent or divergent.

a) \[ \sum_{n=1}^{\infty} \frac{2}{1 + n + n^5}. \]

b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}. \]
Question 6

Determine whether the series is convergent or divergent.

a) \[ \sum_{n=1}^{\infty} n^4 \left( \frac{1}{2} \right)^n. \]

b) \[ \sum_{n=1}^{\infty} \frac{n + 1}{n + 5n^2}. \]

Question 7

Determine whether the series is absolutely convergent, conditionally convergent or divergent.

a) \[ \sum_{n=1}^{\infty} (-1)^n e^{-n}. \]

b) \[ \sum_{n=1}^{\infty} (-1)^n \frac{n + 2}{n + 10}. \]
Question 8

Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{2^n(x - 2)^n}{n^2}.$$ 

Question 9

Express the function $f(x) = \frac{1}{1-x}$ as the power series.
Question 10

Express the function \( f(x) = \frac{2x^4}{x^2 - 4x + 3} \) as the power series.
Question 11

a) Find the series representation for the function \( f(x) = \frac{1}{1+x^3} \).

b) Use the series in a) to write out a series representation for

\[ \int_{0}^{1/2} \frac{1}{1 + x^3} \, dx. \]

(Do NOT compute and add the terms for your series!)
Question 12

A rope that weighs 0.5 kg/m is used to lift 10 kg of water up a well 10 m deep.

a) Find the work needed to lift the rope (only) to the top.

b) Find the work needed to lift the water (only) to the top.

c) Find the total work required to lift both the rope and the water to the top.