MATH 464 (THEORY OF PROBABILITY)  
HOMEWORK 5  

SPRING 2019  
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(1) Given a random variable $X$ with probability mass function

$$f_X(k) = \frac{c}{k(k+1)}, \text{ for } k = 1, 2, \ldots$$

(a) Find the value of $c$.
(b) Find $E(X)$.

(2) Find the mean of $X \sim \text{Geometric}(p)$.

(3) Let $X$ be a discrete random variable with a geometric distribution whose mean is 2.
   (a) Find $P(X \geq 6 | X > 4)$, your answer should not have a summation.
   (b) Let $Z = X + 3$. Find the mean and variance of $Z$.

(4) Can we find a random variable $X$ with $E(X) = 4$ and $E(X^2) = 15$? Explain!

(5) For any random variable $X$, $M_X(t) := E(e^{tX})$ is the Moment generating function (to be discussed in Chapter 4). Find $M_X(t)$ if $X$ is a discrete random variable from
   (a) Bernoulli($p$)   (b) $\text{b}(n, p)$  (c) Geometric($p$)  (d) Poisson($p$).

(6) Give an example of a discrete random variable $X$ such that $E(X^n) = 3^{n-1}$ for $n = 1, 2, \ldots$. Then find $\text{Var}(X)$, and $\text{Var}(X^2)$.

(7) The joint probability mass function of the random variables $X$ and $Y$ is given as $f_{X,Y}(x, y) = c(x + y)$ for $x, y = 0, 1, 2, 3$.
   (a) Find the value of $c$.
   (b) Find $P(X > Y)$.
   (c) Find $P(X + Y = 3)$.