(1) Suppose $X$ has density function $f_X(x) = c(1+x^2)$ for $-1 < x < 1$ and $f_X(x) = 0$ elsewhere.
   
   (a) Find the value of $c$.
   
   (b) Find the distribution function of $X$, i.e., $F_X(x)$.
   
   (c) Sketch the graphs of $f_X(x)$ and $F_X(x)$.
   
   (d) Compute $P(0 < X < 0.5)$.

(2) Consider $f(x) = cx^{-1/2}$ for $x \geq 1$, and $f(x) = 0$ otherwise. Show that there is no value of $c$ that makes $f$ a density function.

(3) Let $X$ be a uniform random variable on the interval $(-1, 1)$. Find the distribution and density functions of $Y = |X|$. Is the distribution of $Y$ in our catalogue of distributions? What is it?

(4) Let $F_1$ and $F_2$ be distribution functions of some random variables, show that for every $0 \leq \alpha \leq 1$, the function
   
   $$F = \alpha F_1 + (1 - \alpha) F_2$$
   
   is a distribution function of some random variable.

(5) Suppose $X$ is an exponential random variable with parameter $\lambda = 1$. Find the distribution and density functions of $Y = \ln(X)$.
   
   **Note:** This is called the double exponential distribution.

(6) For any $\omega > 0$, let
   
   $$\Gamma(\omega) := \int_0^{\infty} x^{\omega-1} e^{-x} dx.$$
   
   Show that if $\omega$ is a positive integer then $\Gamma(\omega) = (\omega - 1)!$

(7) Find the distribution of the so-called “extreme value” density function
   
   $$f(x) = \exp(-x - e^{-x})$$
   
   for $x \in \mathbb{R}$. 