(1) The random variable $Y$ is said to be obtained from the random variable $X$ by \textit{truncation} at the point $a$ if
$$Y(\omega) = \begin{cases} 
X(\omega), & \text{if } X(\omega) \leq a \\
a & \text{if } X(\omega) > a 
\end{cases}$$
Write the distribution of $Y$ in terms of the distribution of $X$.

(2) Given the following density function of a random variable $X$,
$$f_X(x) = \begin{cases} 
\frac{1}{\pi \sqrt{x(1-x)}}, & 0 < x < 1 \\
0 & \text{otherwise.} 
\end{cases}$$
(a) Find the distribution function of $X$, i.e., $F_X(x)$.
(b) Show that $E(X) = \frac{1}{2}$.

(3) The random variable $X$ has density function
$$f(x) = cx(1-x), \quad \text{for } 0 \leq x \leq 1.$$ 
Determine the value of $c$, and find the mean and variance of $X$.

(4) If $Z \sim \mathcal{N}(0,1)$. Find the mean and variance of $Y = e^{2Z}$.

(5) Let $X_1, X_2, \ldots, X_n$ be independent identically distributed (i.i.d.) random variables from $U(0,1)$. Denote $V = \max\{X_1, \ldots, X_n\}$ and $W = \min\{X_1, \ldots, X_n\}$.
(a) Find the distributions and the densities and the distributions of each of $V$ and $W$.
(b) Find $E(V)$ and $E(W)$.

(6) Let $Z \sim \mathcal{N}(0,1)$.
(a) Find the density of $Y = |Z|$.
(b) Find $E(Y)$.

(7) Suppose $X \sim \mathcal{N}(0,1)$. Use integration by parts to show that $E(X^k) = (k-1)E(X^{k-2})$. Derive that $E(X^k) = 0$ for all odd $k \geq 1$. Compute $E(X^4)$ and $E(X^6)$. Derive a general formula for $E(X^{2k})$.

(8) Let $X \sim \exp(1)$, find the density function of $Y = (X-2)/(X+1)$.

(9) Find the mean and variance of the Gamma($\lambda, \omega$) distribution.