1. [15 points] **Choose two to solve:** Determine whether each of the following series is convergent or divergent, show your work.

(a) \( \sum_{n=2}^{\infty} \frac{1}{\ln n} \)

(b) \( \sum_{n=1}^{\infty} \frac{5n + 3}{n^2 + 2n - 1} \)

(c) \( \sum_{n=0}^{\infty} \frac{2^n + n}{n^2 + 4} \)
2. [15 points] Choose two to solve: Determine whether each of the following series is convergent or divergent, show your work.

(a) \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \)

(b) \( \sum_{k=1}^{\infty} \frac{3 + \sin k}{k^{1.1}} \)

(c) \( \sum_{n=1}^{\infty} \frac{3^n}{(2n)!} \)
3. [15 points] Find the radius of convergence of the following series

\[ \sum_{n=1}^{\infty} \frac{(2n)!(x-1)^n}{(n!)^2} \]
4. [15 points] Determine whether the following series are absolutely convergent, conditionally convergent, or divergent, show your work.

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-1)^n n}{n + 5} \]
5. [15 points]

(a) Find the Taylor series of $f(x) = \sin(x^2)$ around $x = 0$.
(b) Find the Taylor series of $\int_0^x \sin(t^2) \, dt$ around $x = 0$. 
6. [15 points] Find the first three (nonzero) terms of the Taylor series of

\[ f(x) = \frac{e^{-x}}{1 - x} \]

around \( x = 0 \).
7. [15 points] Consider the differential equation
\[ \frac{dR}{dt} = -2(R - 2)(R + 1)(R + 2). \]
(a) Find all the equilibrium solutions.
(b) Classify the equilibrium solutions as stable or unstable.