[1] Mark each statement as True or False, explain.
(a) If \( w \) is a linear combination of \( u \) and \( v \) in \( \mathbb{R}^n \), then \( u \) is a linear combination of \( w \) and \( v \).
(b) If none of the vectors in \( S = \{v_1, v_2, v_3\} \) is a multiple of one of the other vectors, then \( S \) is linearly independent.
(c) If \( S = \{v_1, v_2, v_3\} \) is a set of linearly dependent vectors, then each vector in \( S \) is in the span of the other vectors.
(d) If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.
(e) If \( A \) is an \( m \times n \) matrix and the equation \( Ax = b \) is consistent for some \( b \), then the columns of \( A \) span \( \mathbb{R}^m \).
(f) If the columns of an \( n \times n \) matrix are linearly independent then they span \( \mathbb{R}^n \).
(g) If \( \{u_1, u_2, u_3\} \) is a basis for a subspace \( W \) of \( \mathbb{R}^n \), then \( \{u_1, u_2, \alpha u_3\} \) is also a basis for \( W \) for every \( \alpha \in \mathbb{R} \).
(h) If \( A^3 = 0 \) then \( \det(A) = 0 \).
(i) If \( A \) is an \( n \times n \) matrix and \( \alpha \in \mathbb{R} \) then \( \det(\alpha A) = \alpha \det(A) \).

[2] Find the value(s) of \( h \) for which the following vectors are linearly independent.
\[
\begin{bmatrix}
1 \\
-1 \\
3
\end{bmatrix}, \begin{bmatrix}
-5 \\
7 \\
8
\end{bmatrix}, \begin{bmatrix}
1 \\
1 \\
h
\end{bmatrix}.
\]

[3] Do the columns of \( A \) span \( \mathbb{R}^3 \)? Explain
\[
\begin{bmatrix}
1 & 3 & 0 \\
0 & 1 & -1 \\
5 & 8 & 7
\end{bmatrix}
\]

[4] Find the rank and nullity of each of the following matrices
\[
\begin{bmatrix}
4 & 5 & 9 & -2 \\
6 & 5 & 1 & 12 \\
3 & 4 & 8 & -3
\end{bmatrix}, \begin{bmatrix}
-3 & 9 & -2 & -7 \\
2 & -6 & 4 & 8 \\
3 & -9 & -2 & 2
\end{bmatrix}.
\]

[5] Suppose that \( A \) is a \( 3 \times 5 \) matrix with \( \text{rank}(A) = 3 \), what is the \( \text{dim null}(A) \)?
If the subspace of all solutions of $Ax = 0$ has a basis consisting of three vectors and if $A$ is a $5 \times 7$ matrix, what is $\text{rank}(A)$?

If possible, construct a $3 \times 4$ matrix $A$ such that $\text{dim \ null}(A) = 2$.

If $A$ is a $6 \times 8$ matrix, what is the smallest possible dimension of $\text{null}(A)$?

Let $A$ be a $5 \times 6$ matrix, and let $\mathbf{b}$ be any vector in $\mathbb{R}^5$. What you can say about the consistency of the system $A\mathbf{x} = \mathbf{b}$ if $\text{rank}(A) = 5$? and if $\text{rank}(A) = 4$? Explain.

Find the inverses of the following matrices

$$A = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 5 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Find the determinant of the matrices

$$\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{bmatrix}$$

Let $A$ be a $4 \times 4$ matrix, with $\det(A) = -3$ and $\det(B) = 4$. Find the following

(a) $\det(AB)$.
(b) $\det(2A)$.
(c) $\det(A^T B A)$.
(d) $\det(A^3 B^{-1})$.

Given that

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

with $\det(A) = 3$, Find each of the following (explain)

(a) $\det \left( 2A^{-1}(A^T)^2 \right)$.
(b) $\begin{vmatrix} a + 2d & b + 2e & c + 2f \\ d + g & e + h & f + i \\ g & h & i \end{vmatrix}$.
(c) $\begin{vmatrix} 2a & 2b & 2c \\ 3a + 4d & 3b + 4e & 3c + 4f \\ -g & -h & -i \end{vmatrix}$.
(d) $\begin{vmatrix} e & d & f \\ b & a & c \\ -h & -g & -i \end{vmatrix}$.