1. [20 points] Row reduce each of the following matrices into the row reduced echelon form

\[
A = \begin{bmatrix}
1 & 4 & 5 & -9 \\
-1 & -2 & -1 & 3 \\
-2 & -3 & 0 & 3 \\
0 & -3 & -6 & 4
\end{bmatrix}, \quad B = \begin{bmatrix}
1 & -2 & -1 & 3 \\
1 & 8 & -2 & 0
\end{bmatrix}
\]
2. [10 points] Let $A = \begin{bmatrix} 0 & 1 & 4 \\ 2 & -3 & 2 \\ 4 & -8 & 12 \end{bmatrix}$, let $b = \begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix}$, and let $W$ be the span of the columns of $A$.

(a) Is $b \in W$? Explain.

(b) Show that the third column of $A$ is in $W$.

3. [15 points] Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined as

$$T(x_1, x_2) = (x_1 - 2x_2, x_1, x_1 + x_2)$$

for every real numbers $x_1, x_2 \in \mathbb{R}$. 
4. [15 points] Consider the system of linear equations that correspond to the following augmented matrix

\[
\begin{bmatrix}
  2 & 2 & 5 \\
  -5 & h & k
\end{bmatrix}
\]

(a) Find all values of \( h \) so that the system has a **unique** solution.
(b) Find all values of \( h \) such that the columns of the coefficient matrix are **linearly dependent**.
(c) Find all values of \( h \) and \( k \) such that the system has **no solution**.

5. [10 points] Let \( A \) be a \( 3 \times 5 \) matrix. Suppose that the fifth column in the row reduced echelon form of \( A \) is the vector 

\[
\begin{bmatrix}
  0 \\
  0 \\
  1
\end{bmatrix}
\]

(a) Describe geometrically the solution set of the homogeneous matrix equation \( Ax = 0 \).
(b) Describe geometrically the solution set of the matrix equation \( Ax = a_2 + a_3 \), where \( a_2 \) and \( a_3 \) are the second and third columns of \( A \).
6. [15 points] Describe all solutions of $Ax = 0$ in **vector form**, where $A$ is row equivalent to the following

$$
\begin{bmatrix}
1 & -4 & -2 & 0 & 3 & -5 \\
0 & 0 & 1 & 0 & 0 & -2 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

7. [15 points] Mark each statement as **True** or **False**, explain.

- ( ) The columns of any $4 \times 3$ matrix are linearly independent.

- ( ) Every linear transformation is a matrix transformation.

- ( ) If $A$ is $2 \times 5$ matrix and $T$ is the matrix transformation $Tx = Ax$, then the domain of $T$ is $\mathbb{R}^5$ and its range is subset of $\mathbb{R}^2$.

- ( ) If $A$ is $3 \times 2$ matrix, then the transformation $x \mapsto Ax$ cannot be onto.

- ( ) The map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x_1, x_2) = (0, 0)$ for all $x_1, x_2 \in \mathbb{R}$ is a linear transformation.