Submit questions 2 and 7 for grading.
- Due on: Friday 05-24-2019.
- You will have a quiz from the following questions on Monday 05-20-2019.

1. The empty set fails to satisfy only one of the requirements of Vector Spaces. Which one?

2. Let $F[z]$ be the vector space of all polynomials with coefficients from the field $F$. Let 
$$W = F[z] \times F^2 = \{(q, u); \ q \in F[z], u \in F^2\}.$$ 
Equip $W$ with a vector addition (+) and a scalar multiplication (·) in such a way that $W$ becomes a vector space, and prove that $W(\cdot, +)$ is a vector space over $F$.

3. Suppose $U_1$ and $U_2$ are subspaces of $V$. Prove that the intersection $U_1 \cap U_2$ is a subspace of $V$.

4. Prove that the union of two subspaces of $V$ is a subspace of $V$ if and only if one of the subspaces is contained in the other.

5. Prove or give a counterexample: if $U_1, U_2, W$ are subspaces of $V$ such that 
$$U_1 + W = U_2 + W$$
Then $U_1 = U_2$.

6. Prove or give a counterexample: if $U_1, U_2, W$ are subspaces of $V$ such that 
$$U_1 \oplus W = U_2 \oplus W$$
Then $U_1 = U_2$.

7. Suppose 
$$U = \{(x, x, y, y) \in F^4 : x, y \in F\}.$$ 
Find a subspace $W$ of $F^4$ such that $F^4 = U \oplus W$.

8. Consider the vector space $V = \mathbb{R}^\mathbb{R}$ of functions functions $f : \mathbb{R} \to \mathbb{R}$. A function $f : \mathbb{R} \to \mathbb{R}$ is called periodic if there exists a positive number $p$ such that $f(x) = f(x + p)$ for all $x \in \mathbb{R}$. Is the set of periodic functions from $\mathbb{R}$ to $\mathbb{R}$ a subspace of $V$? Explain.

9. Consider the vector space $V = \mathbb{R}^\mathbb{R}$ of functions $f : \mathbb{R} \to \mathbb{R}$. Let $U_o \subset V$ and $U_e \subset V$ be the sets of odd and even continuous functions respectively.
- Recall: $f : \mathbb{R} \to \mathbb{R}$ is odd if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$, and is even if $f(-x) = f(x)$ for all $x \in \mathbb{R}$.
  a) Show that for any $f \in V$, 
  $$g(x) = \frac{f(x) + f(-x)}{2}$$ 
is even, and 
  $$h(x) = \frac{f(x) - f(-x)}{2}$$ 
is odd.
  b) Prove that $U_o$ and $U_e$ are subspaces of $V$.
  c) Show that $V = U_o \oplus U_e$. 