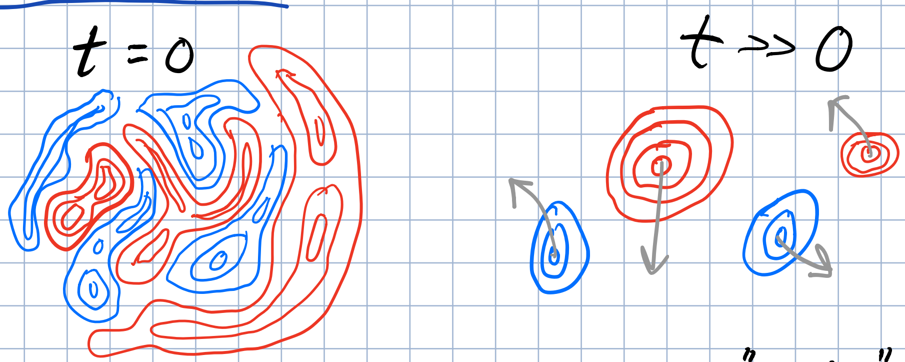
POINT (or BLOB) VORTEX INTEGRABILITY
$\Longleftrightarrow$ LONG-TIME 2D EULER

Basic idea:

On closed 2-manifold M, the long-time behaviour of Euler's equations for generic (smooth) initial data is (in a large part of phase space) determined by integrability properties of low dim PV (or blob) dynamics on M

Illustration:



vortex continuum

vortex "blobs" (due to mixing between equal sign vortex regions)

Question:

i) When (or where in phase space) can we expect vortex blob "asymptotics"

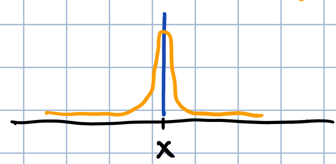
ii) How many vortex blobs do we eventually end up with?

iii) Can we (approx.) describe the motion of the blobs?

Conjecture* (M. & Viviani 2020)

1. N_{blobs} = max number of PV on M for which dynamis is integrable (given the macroscopic variables; energy, circulation, momentum)

2. Center of mass of blobs described by "blob vortex dynamics" where $\delta_x \Leftarrow\Rightarrow K_x$



*DISCLAIMER:
Clearly not very concrete. Part of the problem is to make the question precise

Physical mechanism (heuristic description):

Assumptions:

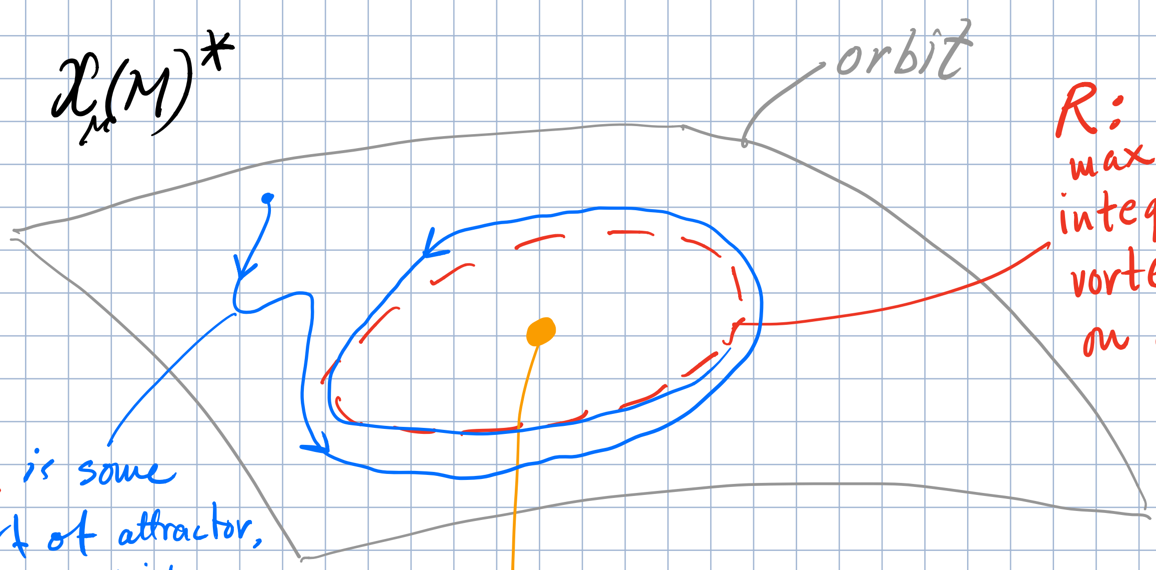
1. Small equal-sign vortex formations merge to larger ones
2. Well-separated blobs interact approx. by blob vortex dynamics

The "algorithm" for long-time behaviour:

Large nr of blobs move chaotically and therefore merging occur sooner or later
UNTIL: blob dynamics is integrable
$\Longrightarrow$ quasi-periodicity prevents further mixing

Motto: integrability acts as a barrier in phase space, preventing further mixing

Geometric picture:



R: max integrable blob vortex configurations on orbit

R is some sort of attractor, but in which sense?

"almost" stationary solution predicted by statistical mechanics theories are blocked by R

(Onsager, Kraichnan, Miller, Robert & Sommeria, etc)

Numerical support of conjecture:

See: slides.com/kmodin/quant-euler

References:

- Onsager, Statistical hydrodynamics (1949)
- Kraichnan, Statistical dynamics of 2D flow (1975)
- Shnirelman, On the long time behavior of fluid flow (2013)
- Marchioro & Pulvirenti, Springer book (2012)
- M. & Viviani, J. Fluid Mech. (2020)