

MAT244 – Fall 2016 – Term Test 1

Problem 1 Find the solution of the following problem

$$\begin{cases} y' + 4y = xe^x \\ y(0) = 1 \end{cases} .$$

Problem 2 Solve the following problem

$$\begin{cases} y(x+y) + (xy+1)y' = 0 \\ y(0) = e \end{cases} ,$$

given that the equation has an integrating factor of the form $\mu = \mu(y)$.

Problem 3 Consider the following Bernoulli problem

$$\begin{cases} y' + y = r(x)y^2 \\ y(0) = 1 \end{cases} ,$$

where $r(x)$ is

$$r(x) = \begin{cases} 1 & x \leq 1 \\ 0 & x > 1 \end{cases} .$$

1. Find the solution for $x \leq 1$.
2. Assuming that the solution is continuous at $x = 1$, use the result of part (1) to find the solution to the problem for $x > 1$.

Problem 4 Consider the equation $y' = y - y^3$.

- i. Find all equilibria of the equation.
- ii. Draw the phase line and determine stability of each equilibrium.
- iii. Sketch in the xy -plane the graphs of solution satisfying conditions $y(0) = 2$, $y(0) = -2$, and $y(0) = \frac{3}{4}$.

Problem 5 Solve the following equation

$$y' = \frac{y-x}{y+x+2}.$$

Hint: you can use the axis translation $X = x + 1$ and $Y = y + 1$.