MAT244H1F1: Introduction to Ordinary Differential Equations

First Midterm, Question 4 Solution

Problem 1 *Consider the equation $y' = y - y^3$.*

1. *Find all equilibria of the equation.*
2. *Draw the phase line and determine the stability of each equilibrium.*
3. *Sketch in the xy-plane the graph of solutions satisfying conditions $y(0) = 2, y(0) = -2$ and $y(0) = \frac{3}{4}$.*

To find the equilibrium points we equate the derivative with 0 and solve, in this case we have

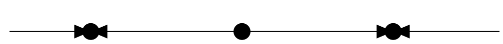
$$0 = y - y^3 = y(1 - y^2) = y(1 - y)(1 + y),$$

and hence the equilibrium points are $0, 1$ and -1.

To determine the stability we study the sign of the derivative in the regions around the equilibrium points, that is, in the intervals $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$ and $(1, \infty)$. We recall that the sign of the derivative is constant in each of these intervals. We have the following table

	y	$1-y$	$1+y$	$y-y^3$
$(-\infty, -1)$	$-$	$+$	$-$	$+$
$(-1, 0)$	$-$	$+$	$+$	$-$
$(0, 1)$	$+$	$+$	$+$	$+$
$(1, \infty)$	$+$	$-$	$+$	$-$

In this table a $+$ sign means that term is positive and $-$ sign means it is negative. From we know that solutions approach -1 and 1 and move away from 0, hence 1 and -1 are stable and 0 is unstable. We deduce that the phase line looks like



Finally we show the solutions to these equations (together with another solution starting at point $y(0) = -\frac{3}{4}$):