

Total: 40 points. No aids allowed!

1. (10 pts) Solve the initial value problem

$$\begin{cases} y'' - y' - 6y = 6, \\ y(0) = 1, \\ y'(0) = 1. \end{cases}$$

2. (10 pts)

- (a) Find a solution of the equation

$$x^2 y'' - x(x+2)y' + (x+2)y = 0$$

of the form  $y = ax + b$ , where  $a$  and  $b$  are constants.

- (b) Find the general solution of the equation

$$x^2 y'' - x(x+2)y' + (x+2)y = 2x^3 e^{2x}.$$

3. (10 pts)

- (a) Find the general solution of

$$y^{(6)} - y^{(2)} = 0.$$

- (b) Find the general solution of

$$y^{(6)} - y^{(2)} = x.$$

4. (10 pts)

- (a) Solve the equation

$$y''' + y' = \cos(kx),$$

where  $k$  is a constant.

- (b) For which values of  $k$  does the equation in part (a) have an unbounded solution?