MAT337H1, Introduction to Real Analysis: additional recommended problems for Feb 1 class

1. Let
$$s_n = \sum_{k=1}^{n} \frac{\sin(k)}{k^2}.$$
Show that the sequence s_n is convergent by showing that it is Cauchy.

2. Let a_n and b_n be Cauchy sequences. Show that the sequence
$$a_1, b_1, a_2, b_2, \ldots$$
is Cauchy if and only if
$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n.$$

3. Which of the following definitions of a Cauchy sequence are correct (i.e., equivalent to the one given in class / textbook)?

   (a) A sequence a_n is Cauchy if for any real number $\varepsilon > 0$ there exists a positive integer N such that for any integer $n \geq N$ and any non-negative integer k we have $|a_{n+k} - a_n| < \varepsilon$.

   (b) A sequence a_n is Cauchy if for any real number $\varepsilon > 0$ and any non-negative integer k there exists a positive integer N such that for any integer $n \geq N$ we have $|a_{n+k} - a_n| < \varepsilon$.

   If any of these definitions is wrong (i.e., not equivalent to the standard one), provide an example demonstrating this.