MAT337H1, Introduction to Real Analysis: additional recommended problems for Feb 15 class

1. In class we proved that if f is a continuous function on $[a, b]$, and ξ is a number such that $f(a) < \xi < f(b)$, then there is $c \in [a, b]$ such that $f(c) = \xi$. We defined c by the formula $c = \sup\{x \in [a, b] \mid f(x) < \xi\}$. Then we showed that $f(c) = \xi$ by considering two cases $f(c) > \xi$ and $f(c) < \xi$ and drawing a contradiction in both cases. However, our argument does not work if $c = a$ in the first case or $c = b$ in the second case. Show that both these situations are in fact impossible.

2. Let f be a strictly increasing continous function on $[a, b]$. Show that f^{-1} is continuous at the endpoints $f(a)$ and $f(b)$ of its domain.

3. Let X, Y, Z be arbitrary sets, and let $f \colon Y \to Z, g \colon X \to Y$ be bijections. Show that the composition $f \circ g \colon X \to Z$ is a bijection and that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$ (socks-shoes property).