

**MAT337H1, Introduction to Real Analysis: additional recommended problems
for Feb 3 class**

1. Show, using the definition of continuity, that the following functions are continuous at every point where they are defined.

(a) $f(x) = x^3$.

(b) $f(x) = \frac{1}{x}$.

(c) $f(x) = \sqrt{x}$. (Recall that for any $x \geq 0$, the number \sqrt{x} can be defined using supremums. Here you do not need the exact definition, but only need to know that \sqrt{x} satisfies $(\sqrt{x})^2 = x$).

2. Is it possible to define the value of the function $f(x) = \frac{1}{x}$ at $x = 0$ in such way that this function becomes continuous? If yes, find this value and prove the continuity of the resulting function. If not, prove that the resulting function is always discontinuous no matter how we define it at 0.