

MAT337H1, Introduction to Real Analysis: Final exam coverage

- You need to know the definitions of and be able to use the following things:
 - upper and lower bound of a subset of real numbers;
 - bounded above or below subset of real numbers;
 - least upper bound (sup) and greatest lower bound (inf) for a subset of real numbers;
 - limit of a sequence of real numbers, convergent sequence;
 - (strictly) monotone sequence;
 - subsequence;
 - Cauchy sequence;
 - limit of a function;
 - continuous function;
 - increasing, decreasing, and monotone functions;
 - differentiable function, derivative;
 - uniformly continuous function;
 - Riemann integrable function on a closed interval;
 - Riemann integral of a function on a closed interval;
 - partition of a closed interval;
 - mesh of a partition;
 - evaluation sequence for a partition;
 - Riemann sum;
 - lower and upper sums;
 - refinement of a partition;
 - metric space;
 - convergent sequence in a metric space;
 - Cauchy sequence in a metric space;
 - complete metric space;
 - uniform convergence of a sequence of functions;
 - continuous mapping between metric spaces.

You do not need to cite exactly the definition stated in class. It is fine if you formulate it in your own words.

- Although you will not be asked to formulate or prove theorems discussed in class or in the textbook, these results (as well as techniques used to prove them) might be useful for solving problems. This includes the following results:
 - the least upper bound principle;
 - squeeze theorem;
 - any convergent sequence is bounded;
 - limit of the sum/difference/product/ratio theorem;
 - monotone convergence theorem;
 - nested intervals lemma;
 - Bolzano-Weierstrass theorem;
 - equivalence between being Cauchy and being convergent;
 - continuity criterion in terms of sequences;
 - arithmetic operations with functions preserve continuity;
 - composition of continuous functions is continuous;
 - intermediate value theorem;
 - a continuous (strictly) monotone function is a bijection of its domain to its range; its inverse function is also continuous;
 - a continuous function on a closed interval is bounded;
 - a continuous function on a closed interval attains its supremum and infimum;
 - arithmetic properties of differentiation, chain rule;
 - Fermat's theorem, Rolle's theorem, mean value theorem;
 - a function continuous on a closed interval is uniformly continuous on that interval;
 - a function continuous on a closed interval is Riemann integrable on that interval;
 - fundamental theorem of calculus;
 - any convergent sequence in a metric space is Cauchy;
 - continuity criterion in terms of sequences for mappings between metric spaces.