MAT337H1, Introduction to Real Analysis: additional recommended problems for Jan 11 class

1. Assume that $a, b \in \mathbb{R}$, $a, b > 0$, $a \neq b$. Let $a_0.a_1\ldots$ and $b_0.b_1\ldots$ be decimal expansions of a and b respectively. Let also i be a minimal non-negative integer such that $a_i \neq b_i$. Then

   (a) if $a_i > b_i$, we say that $a > b$;

   (b) if $a_i < b_i$, we say that $a < b$.

   You need to show that this definition is unambiguous, i.e. that the so defined notion of a being greater (or less) than b does not depend on the choice of their infinite decimal expansions.

2. Prove that the set $\{x \in \mathbb{Q} \mid x > 0, x^2 < 2\}$ is bounded above but does not have a supremum in $\mathbb{Q}$.