MAT337H1, Introduction to Real Analysis: recommended problems for Jan 13 class

1. Show that $\inf\{x \in \mathbb{R} \mid x > -1\} = -1$.

2. Let S be a bounded below set containing at least one negative number. Recall the procedure for constructing $\inf S$ for such a set. Let $-m$ be a negative integer lower bound for S (why does such a lower bound exist?). Consider the integers $0, -1, \ldots, -(m-1)$. Find the smallest one which is not a lower bound for S (why does it exist?). Denote this integer by $-a_0$ (where a_0 is a non-negative integer). Then consider the numbers $-a_0.0, -a_0.1, \ldots, -a_0.9$. Find the smallest one which is not a lower bound for S, and denote it by $-a_0.a_1$. Then consider the numbers $-a_0.a_1 0, -a_0.a_1 1, \ldots, -a_0.a_1 9$. Find the smallest one which is not a lower bound for S, and denote it by $-a_0.a_1a_2$. Continuing this procedure, we obtain an infinite decimal expansion $-a_0.a_1a_2\ldots$. Show that the corresponding real number is the infinum of S.

3. Apply the procedure described in Problem 2 to the set from Problem 1. What infinite decimal expansion do you get? How does it agree with the statement of Problem 1?

4. For any subset $S \subset \mathbb{R}$, set $-S = \{-x \mid x \in S\}$. Show that if S is bounded above, then $-S$ is bounded below. Also show that for $S \subset \mathbb{R}$ bounded above we have

$$\sup S = -\inf(-S).$$