**MAT337H1, Introduction to Real Analysis: recommended problems for Mar 17 class**

1. Show, using the definition of the Riemann integral in terms of partitions (and not using the fundamental theorem of calculus) that

$$\int_a^b \frac{1}{x}dx = \int_{ka}^{kb} \frac{1}{x}dx$$

for any positive numbers $a, b, k$.

2. For $x > 0$, define a function $\ln(x)$ by the formula

$$\ln(x) = \int_1^x \frac{1}{t}dt.$$

Show, using this definition, that $\ln(xy) = \ln(x)+\ln(y)$. Hence show that ln is a bijection from positive numbers to all real numbers.

3. Define a function exp from real numbers to positive real numbers as the inverse of the function ln. Show that $\exp' = \exp$.

4. The functions ln and exp provide us with an easy way to define $x^y$ even when $y$ is irrational. Namely, for positive $x$ and any real $y$ we set $x^y = \exp(y \ln x)$. Show that for rational $y$ this coincides with our earlier definition $x^{\frac{m}{n}} = \sqrt[n]{x^m}$.