MAT337H1, Introduction to Real Analysis: Test 2 coverage

Term Test 2 will be based on the material covered in Jan 6 - Mar 1 classes, with emphasis on the material covered in Feb 3 - Mar 1 classes (see Sections 5.1, 5.3, 5.4, 5.6, 5.7, 6.1, 6.2 of the textbook). There will be proof questions and definition questions.

- Proof questions will be similar to ones from the recommended problems list.
- For definitions, you need to be able to define the following:
  - all definitions covered by Test 1;
  - limit of a function (this is Def. 5.1.1 from the textbook, with $m = 1$, $n = 1$, and S an open interval containing the point a; ignore the condition "if a is a limit point");
  - continuous function (Def. 5.1.2 with $m = 1$ and $n = 1$);
  - increasing, decreasing, and monotone functions (Def. 5.7.1);
  - differentiable function, derivative (Def. 6.1.1).

  You do not need to cite exactly the definition stated in class. It is fine if you formulate it in your own words.
- Although you will not be asked to formulate or prove theorems discussed in class or in the textbook, these results might be useful for solving problems (and solutions to many problems are similar to proofs of theorems discussed in class!). This includes the following results:
  - all theorems covered by Test 1;
  - continuity criterion in terms of sequences (parts (1) and (2) of Thm. 5.3.1, with $n = 1$ and $m = 1$);
  - arithmetic operations with functions preserve continuity (Thm. 5.3.2 with $n = 1$ and $m = 1$);
  - composition of continuous functions is continuous (Thm. 5.3.5 with $n = 1$, $m = 1$, and $l = 1$);
  - intermediate value theorem (Thm. 5.6.1);
  - a continuous (strictly) monotone function is a bijection of its domain to its range; its inverse function is also continuous (Thm. 5.7.6);
  - a continuous function on a closed interval is bounded (not in the textbook, but follows from Thm. 5.4.3 with $n = 1$ and C a closed interval);
  - a continuous function on a closed interval attains its supremum and infinum (Thm. 5.4.4 with $n = 1$ and C a closed interval);
  - arithmetic properties of differentiation, chain rule (the chain rule is Thm. 6.1.6, arithmetic properties are stated in Exercises A, F, H for Section 6.1);
  - a function $f(x)$ is differentiable at x_0 if it can be written as $f(x_0) + (x - x_0)\phi(x)$ where ϕ is continuous at x_0 (parts (1) and (3) of Cor. 6.1.4);
  - Fermat's theorem (Thm. 6.2.1), Rolle's theorem, mean value theorem (Thm. 6.2.2).