

MATH534A, Final Exam
December 11, 2018

All problems are worth the same number of points.

1. Let $a > b > c > 0$ be real numbers. Find all real numbers $r > 0$ such that the intersection of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

with the sphere

$$x^2 + y^2 + z^2 = r^2$$

is a smooth submanifold of \mathbb{R}^3 .

2. Prove that the map

$$u \mapsto \left(\frac{1}{2} \left(u + \frac{1}{u} \right), \frac{i}{2} \left(u - \frac{1}{u} \right) \right)$$

is an embedding of $\mathbb{C} \setminus \{0\}$ to \mathbb{C}^2 .

3. Prove that $\mathbb{R}\mathbb{P}^4$ is a non-orientable manifold.

4. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be a smooth function such that $df \neq 0$ at points where $f = 0$. Let α be a 1-form on the submanifold $\{f = 0, \frac{\partial f}{\partial y} \neq 0\} \subset \mathbb{R}^2$ given by

$$\alpha = \frac{dx}{\partial f / \partial y}.$$

Show that α extends to a smooth 1-form on the submanifold $\{f = 0\}$.