MATH534A, Problem Set 1, due Aug 30

All problems are worth the same number of points.

1. Problem 1-1 from the textbook (p. 29).
2. Problem 1-2 from the textbook (p. 30).
3. For the sphere
$$S^n = \{(x^1, \dots, x^{n+1}) \in \mathbb{R}^{n+1} \mid \sum_{i=1}^{n+1} (x^i)^2 = 1\},$$
let
$$U_+^i = \{(x^1, \dots, x^{n+1}) \in S^n \mid x^i > 0\}, \quad U_-^i = \{(x^1, \dots, x^{n+1}) \in S^n \mid x^i < 0\}.$$
Define also $\phi_\pm^i : U_\pm^i \to \mathbb{R}^n$ by
$$\phi_\pm^i(x^1, \dots, x^{n+1}) = (x^1, \dots, x^{i-1}, x^{i+1}, \dots, x^{n+1}).$$
Compute the transition maps for the charts $(U_\pm^i, \phi_\pm^i)$. Hence show that S^n, covered by these charts, is a smooth manifold.
4. Let X be a 1-dimensional manifold covered by two charts with domains U and V respectively. Let also $u: U \to \mathbb{R}$ and $v: V \to \mathbb{R}$ be the associated coordinates. Assume that the transition map is $v = e^u$. Find a cover of $X \times X$ by coordinate charts and compute the corresponding transition maps.
5. Show that the circle S^1 cannot be covered by one chart.