MATH534A, Problem Set 4, due Sep 27

All problems are worth the same number of points.

1. Let $\phi\colon M \to N, \psi\colon L \to M$ be smooth maps between smooth manifolds. Use three different definitions of the differential of a smooth map (in terms of velocity vectors of curves, in terms of differential operators, and in terms of the Jacobian matrix) to verify that $d(\phi \circ \psi) = d\phi \circ d\psi$ (i.e. you should present three different proofs).

2. Let $S^2 \subset \mathbb{R}^3$ be the unit sphere centered at the origin, and let $\phi_\alpha\colon S^2 \to S^2$ be rotation of the sphere by an angle α about the z-axis. Compute the differential of ϕ_α at the north pole.

3. Problem 3-5 from the textbook.

4. Let M be a smooth manifold. Show that a chart (U, ϕ) on M is smooth (i.e. it can be included in a smooth atlas defining the given smooth structure on M) if and only if $\phi\colon U \to \phi(U)$ is a diffeomorphism.

5. (Notation corrected on Sep 20) Let M be a smooth manifold of dimension n, and N be its smooth submanifold of dimension m. By definition, this means that for every $p \in N$ there exists a smooth chart (U_p, ϕ_p) in M such that $p \in U_p$, and $\phi_p(U_p \cap N) = \phi_p(U_p) \cap \{x^{m+1} = 0, \ldots, x^n = 0\}$. Show that the collection $(U_p \cap N, \phi_p\,|_{U_p \cap N})$, where $p \in N$, is a smooth atlas on N which turns N into a smooth manifold of dimension m.