**MATH534A, Exam 1**
**October 2, 2018**

All problems are worth the same number of points.

1. Give a definition of a smooth manifold. You are only allowed to use terminology from multivariable calculus and point-set topology. Everything else should be explicitly defined.

2. Let $S^2$ be the standard unit sphere, $U \subset S^2$ be the northern hemisphere, and $p$ be a point in $U$. Let also $(x_1, x_2)$ be coordinates in $U$ given by $x_1(x, y, z) = x$, $x_2(x, y, z) = y$, and let $\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}$ be the associated basis of $T_pM$. Compute $\mathrm{d}i(\frac{\partial}{\partial x_1})$, where $i\colon S^2 \to \mathbb{R}^3$ is the inclusion map, and $\mathrm{d}i$ is its differential (at $p$).

3. Let $S^2$ be the standard unit sphere, $n \in S^2$ be its north pole, and let $\sigma\colon S^2 \setminus \{n\} \to \mathbb{C}$ be the stereographic projection from $n$ (here we identify $\mathbb{R}^2$ and $\mathbb{C}$). Define a map $\phi\colon S^2 \to \mathbb{CP}^1$ by

$$\phi(p) = \begin{cases} [\sigma(p) : 1] & \text{if } p \neq n, \\ [1 : 0] & \text{if } p = n. \end{cases}$$

   Prove that $\phi$ is a diffeomorphism.

4. Prove that the set $\{(x, y, z) \in \mathbb{RP}^2 \mid xy = z^2\}$ is a smooth submanifold of $\mathbb{RP}^2$.