MATH534A, Exam 2
November 6, 2018

All problems are worth the same number of points.

1. Show that the map $S^2 \to \mathbb{R}^4$ given by $(x, y, z) \mapsto (xy, xz, y^2 - z^2, 2yz)$ is an immersion.

2. Consider the map $\phi \colon \mathbb{R}^2 \to S^2$ given by

$$x = \cos(u)\cos(v), \quad y = \cos(u)\sin(v), \quad z = \sin(u).$$

   Find the 1-form $\phi^*\alpha$, where α is the restriction to S^2 of the 1-form $xdy - ydx$.

3. Let M be a compact manifold, and let $C \subset M$ be its closed subset. Let also v be a smooth vector field defined on some open set $U \supset C$. Show that there exists a smooth vector field $\hat{v}$ on M such that $\hat{v}|_C = v$.