1. Let \( X \) and \( Y \) be homotopy equivalent topological spaces. Assume also that \( X \) has the following property: any two continuous maps \( S^2 \to X \) are homotopic to each other. Prove that \( Y \) has the same property.

2. Recall that the Möbius band is the space obtained from a rectangle by identifying a pair of its opposite sides as shown in the figure:

   \[
   \begin{array}{c}
   \text{a} \\
   \text{a}
   \end{array}
   \]

   Prove that there exists no retraction of the Möbius band to its boundary.

3. Consider the space \( X \) obtained from an octagon by identifying its sides as shown in the figure:

   \[
   \begin{array}{c}
   a \\
   b \\
   a \\
   \text{x_0} \\
   d \\
   c \\
   d \\
   \end{array}
   \]

   The dashed path in the figure determines a loop in \( X \) based at \( x_0 \), and hence an element of \( \pi_1(\mathbb{X}, x_0) \). Is that element equal to the identity?