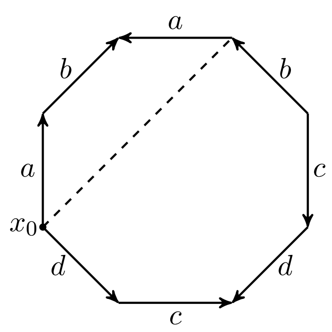
MATH534B, Exam 1
February 26, 2019

1. Let X and Y be homotopy equivalent topological spaces. Assume also that X has the following property: any two continuous maps $S^2 \to X$ are homotopic to each other. Prove that Y has the same property.

2. Recall that the Möbius band is the space obtained from a rectangle by identifying a pair of its opposite sides as shown in the figure:

   Prove that there exists no retraction of the Möbius band to its boundary.

3. Consider the space X obtained from an octagon by identifying its sides as shown in the figure:



   The dashed path in the figure determines a loop in X based at x_0, and hence an element of $\pi_1(X, x_0)$. Is that element equal to the identity?