Symplectic Structure of the Non-Linear Schrödinger Equation

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Abstract

Symplectic and Poisson structures are the basis for interpreting PDEs such as KdV, NLS, Sine-Gordon, etc. as infinite dimensional Hamiltonian systems. This talk is a description of the symplectic structure for the NLS using sources [1] [2] [3]. I discuss the underlying Lie Group and Lie sub-Algebra to define the phase space of a symplectic manifold. Then point out how the symplectic manifold inherits structure from the Lie sub-algebra to turn it into pre-Hilbert space. Then I define a symplectic form to derive the complete symplectic structure. Finally, an example is shown on how to derive the symplectic gradient of a function on the phase space to derive the NLS. The NLS is expressed as a Hamiltonian flow of the form $\dot{u} = (\nabla_s H)_u$ for u in the phase space and H, a function on the phase space.

References

[1] M. Ablowitz and H. Segur. *Solitons and the Inverse Scattering Transform.* Society for Industrial and Applied Mathematics, 1981.

[2] M. A. Ablowitz and P. A. Clarkson. *Solitons, Nonlinear Evolution Equations and Inverse Scattering.* London Mathematical Society Lecture Note Series. Cambridge University Press, 1991.

[3] Richard S. Palais. The symmetries of solitons, 1997.