Random Triangulations of a Riemannian Manifold

Thomas Doehrman

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1 Abstract

One way of computing the Euler characteristic of a compact 2 dimensional manifold M is with the formula

$$\chi(M) = V - E + F$$

where V, E, and F, are the number of vertices, edges, and faces respectively in any triangulation of M. If one selects a triangulation via some random process, V, E, and F become random variables. By linearity of expectation, and since the Euler characteristic does not depend on the choice of triangulation, we obtain:

$$\chi(M) = \mathbb{E}[V] - \mathbb{E}[E] + \mathbb{E}[F]$$

If M is has a Riemannian metric, one can define a sequence of random triangulations for which, in the limit, this equation becomes exactly the Gauss Bonnet theorem (without boundary):

$$\chi(M) = \int_M k \, dA.$$

This non-standard approach to proving Gauss Bonnet is due to Leibon [1].

References

[1] Gregory Leibon. "Random Delaunay triangulations and metric uniformization". In: *International Mathematics Research Notices* (Dec. 2000). DOI: 10.1155/S1073792802201166.