QRT Mappings and Rational Surfaces

Brandon Tippings

In the late 1980s, Quispel, Roberts, and Thompson introduced an 18-paremeter family of integrable mappings, now referred to as the QRT mappings. Orbits of these mappings are confined to invariant biquadratic level sets, which are easily computed from the parameters of the system. Furthermore, the dynamics along these level sets are governed by a series of involutions. We will illustrate these level sets and involutions with a concrete example of q-Painlevé VI. We will also show that the pencil of invariant biquadratics has connections to a rational surface associated with these mappings. These rational surfaces are obtained by successive blowups of $\mathbb{P}^1 \times \mathbb{P}^1$.