1. (5 points) What is a basis of a vector space \( V \)?
   
   a linearly independent spanning set/list of \( V \)

2. (10 points) Let \( V = \{ p \in \mathcal{P}_2(F) : p(0) = 0 \} \), where \( \mathcal{P}_2(F) \) denotes the vector space of all polynomials with coefficients in \( F \) and degree at most 2.

   (a) Show that \( V \) is a subspace of \( \mathcal{P}_2(F) \). Show all your work.

   Assume \( \forall p, q \in V \)
   
   \( \Rightarrow (p + q)(0) = p(0) + q(0) = 0 \)
   
   \( \Rightarrow p + q \in V \)

   Assume \( \forall p \in V, \forall a \in F \)
   
   \( \Rightarrow ap(0) = a(p(0)) = a \cdot 0 = 0 \Rightarrow ap \in V \)

   (b) Verify that \( t^2, t - t^2 \) is a basis of \( V \). Show all your work.

   1) \( at^2 + b(t - t^2) = bt + (a - b)t^2 = 0 \)
   
   \( \Rightarrow a - b = 0 \Rightarrow \{ a = 0 \} \)

   \( \Rightarrow t^2, t - t^2 \) lin indep

   2) \( \forall p = a_0 + a_1 t + a_2 t^2 \in V \)

   \( p(0) = a_0 = 0 \Rightarrow p = a_1 t + a_2 t^2 = a_1(t - t^2) + (a_2 + a_1) t^2 \in \text{Span}\{t, t - t^2\} \Rightarrow \text{Span}\{t, t - t^2\} = V \)

   (c) Is \( t + 2t^2, 3t + 4t^2, 5t + 6t^2 \) a basis of \( V \)? Why? Show all your work.

   No, by (b), \( V \) has a spanning list of length 2

   \( \Rightarrow t + 2t^2, 3t + 4t^2, 5t + 6t^2 \) cannot be lin indep.

   * Alternatively, use the fact that the length of a basis of \( V = \text{dim} V = \frac{1}{2} \)