Math 466 - Practice Problems for Exam 2

1. Suppose that $X_1, \ldots, X_n$ for a random sample from a uniform distribution on the interval $[0, \theta]$. Consider the hypothesis

$$H_0 : \theta \geq 1 \quad \text{versus} \quad H_1 : \theta < 1$$

Let$T(X) = \max\{X_1 \ldots, X_n\}$ and consider the critical region $C = \{\mathbf{x}; T(\mathbf{x}) \leq 2/3\}$.

   (a) Compute the power function for this test

   (b) Determine the size of this test.

2. The lifetime (in years) of a particular brand of car battery has a mean of μ and a standard deviation of σ.

   (a) Suppose the population mean μ is 4.3 and the population standard deviation σ is 4.8. The lifetimes of a sample of n batteries are found. The probability the sample mean $\bar{X}_n$ is between 4.1 and 4.5 is approximately $P(a \leq Z \leq b)$ where Z is a standard normal random variable. Find the values of a and b. (Your answers should depend on n.)

   (b) Now suppose that the population mean is unknown, but the population standard deviation is still 4.8. For a sample of 400 batteries, the sample mean is $\bar{X}_n = 4.1$. Find a 95% confidence interval for the population mean μ.

3. Suppose it is known from recent studies that the average systolic blood pressure in American men over 60 is 130 (mm Hg). The claim is made that a new drug reduces blood pressure in such men within three months. To test this claim a random sample of 50 men from this group are treated with the drug for three months. Then their blood pressures are measured. The sample mean is found to be $\bar{X}_n = 128.0$ and the sample variance is $s^2 = 49.7$. Denote by μ the population mean systolic pressure after treatement. In other words, if we gave all American men over 60 the treatment for three months, μ would be the mean blood pressure of this population.

   (a) State the appropriate null hypothesis.

(b) State the appropriate alternative hypothesis. (You may assume there is no reason to expect the drug to raise blood pressure.)

(c) Specify what the test is if we want a significance level of 0.05, and decide is you accept the null or alternative hypothesis.

(d) Specify what the test is if we want a significance level of 0.01, and decide is you accept the null or alternative hypothesis.

$$P(Z < 1.281552) = 0.9, \qquad P(Z < 1.644854) = 0.95,$$
$$P(Z < 1.959964) = 0.975, \qquad P(Z < 2.053749) = 0.98,$$
$$P(Z < 2.326348) = 0.99, \qquad P(Z < 2.575829) = 0.995$$

4. We consider the following two populations. Population 1 is all working adults in the US with a college degree. Population 2 is all working adults in the US without a college degree. We consider their annual income in thousands of dollars, and let μ_1 and μ_2 be the means for the two populations. And let σ_1^2 and σ_2^2 be the variances for the two populations. We want to estimate $\mu_1 - \mu_2$, the average increase in salary from a college degree. Samples of size $n_1 = 400$ and $n_2 = 100$ are randomly chosen from the two populations. We find that their sample means and variances are

$$\bar{X}_{1,n_1} = 51.6, \quad s_1^2 = 224, \qquad \bar{X}_{2,n_2} = 27.9, \quad s_2^2 = 63$$

(Remember these are in thousands of dollars, so 51.6 is $\$51,600$.)

(a) $\bar{X}_{1,n_1} - \bar{X}_{2,n_2}$ is the natural estimator for $\mu_1 - \mu_2$. What is the variance of this estimator in terms of σ_1^2 and σ_2^2 ?

(b) Find a 95% confidence interval for $\mu_1 - \mu_2$.

5. The NRA claims that 40% of the US adult population is opposed to gun control legislation. To test this claim against the hypothesis that the percentage is less than 40%, a random sample of 400 US adults is chosen. It is found that 140 of the 400 are opposed to such legislation.

(a) State the null and alternative hypotheses.

(b) Specify what the test is if we want a significance level of 0.05, and decide if you accept the null or alternative hypothesis.

6. A population has unknown mean μ and known variance $\sigma^2 = 400$. We want to test the null hypothesis $\mu = 100$ against the alternative hypothesis $\mu > 100$. We have a large sample with sample mean $\bar{X}_n$. Let

$$Z = \frac{\bar{X}_n - 100}{\sigma/\sqrt{n}}$$

Our test is that we reject the null hypothesis if $Z > 1.645$.

(a) What is the significance level of this test?

(b) Recall that the power is the probability we reject the null hypothesis. It depends on μ. Suppose that $n = 100$. What is the power when $\mu = 105$?

7. A manufacturer of a brand of light bulbs claims that the mean life-time μ of their bulbs is more than one year.

(a) Find an appropriate test with significance level $\alpha = 0.05$ of the null hypothesis $H_0 : \mu = 1$ against the alternative hypothesis $H_a : \mu > 1$ (the manufacturer's claim). You may assume that the sample size is large.

(b) Suppose that a sample of size $n = 40$ has sample mean $\overline{X_n} = 1.5$ and sample variance $s^2 = 1.7$. Would you accept the manufacturer's claim?

(c) Compute the power of the above test if $\mu = 1.3$ and $n = 40$.

8. According to the Hardy-Weinberg formula, a genotype has two alleles A_1 and A_2, with gene frequencies p_1 and p_2, $p_1 + p_2 = 1$ should be in proportions $p_1^2 : 2p_1p_2 : p_2^2$ for respectively homozygous A_1 (A_1A_1), heterozygous (A_1A_2), and homozygous A_2 (A_2A_2) individuals. Test the hypothesis that A_1 and A_2 follow this formula with $p = 0.5$ with

genotype i	A_1A_1	A_1A_2	A_2A_2
O_i	18	60	22