

## A Short Table of Indefinite Integrals

### I. Basic Functions

1.  $\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$
2.  $\int \frac{1}{x} dx = \ln|x| + C$
3.  $\int a^x dx = \frac{1}{\ln a} a^x + C$
4.  $\int \ln x dx = x \ln x - x + C, \quad x > 0$
5.  $\int \sin x dx = -\cos x + C$
6.  $\int \cos x dx = \sin x + C$
7.  $\int \tan x dx = -\ln|\cos x| + C$

### II. Products of $e^x$ , $\cos x$ , and $\sin x$

8.  $\int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C$
9.  $\int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C$
10.  $\int \sin(ax) \sin(bx) dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b$
11.  $\int \cos(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, \quad a \neq b$
12.  $\int \sin(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b$

### III. Product of Polynomial $p(x)$ with $\ln x$ , $e^x$ , $\cos x$ , $\sin x$

13.  $\int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1, \quad x > 0$
14.  $\int p(x) e^{ax} dx = \frac{1}{a} p(x) e^{ax} - \frac{1}{a} \int p'(x) e^{ax} dx$   
 $= \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \dots$   
 (+ - + - ...) (signs alternate)
15.  $\int p(x) \sin ax dx = -\frac{1}{a} p(x) \cos ax + \frac{1}{a} \int p'(x) \cos ax dx$   
 $= -\frac{1}{a} p(x) \cos ax + \frac{1}{a^2} p'(x) \sin ax + \frac{1}{a^3} p''(x) \cos ax - \dots$   
 (- + - + ...) (signs alternate in pairs after first term)
16.  $\int p(x) \cos ax dx = \frac{1}{a} p(x) \sin ax - \frac{1}{a} \int p'(x) \sin ax dx$   
 $= \frac{1}{a} p(x) \sin ax + \frac{1}{a^2} p'(x) \cos ax - \frac{1}{a^3} p''(x) \sin ax - \dots$   
 (+ + - - + + ...) (signs alternate in pairs)

### IV. Integer Powers of $\sin x$ and $\cos x$

17.  $\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx, \quad n \text{ positive}$
18.  $\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx, \quad n \text{ positive}$
19.  $\int \frac{1}{\sin^m x} dx = \frac{-1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} dx, \quad m \neq 1, m \text{ positive}$
20.  $\int \frac{1}{\sin x} dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$
21.  $\int \frac{1}{\cos^m x} dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} dx, \quad m \neq 1, m \text{ positive}$
22.  $\int \frac{1}{\cos x} dx = \frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C$
23.  $\int \sin^m x \cos^n x dx$ : If  $m$  is odd, let  $w = \cos x$ . If  $n$  is odd, let  $w = \sin x$ . If both  $m$  and  $n$  are even and non-negative, convert all to  $\sin x$  or all to  $\cos x$  (using  $\sin^2 x + \cos^2 x = 1$ ), and use IV-17 or IV-18. If  $m$  and  $n$  are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both  $m$  and  $n$  are even and negative is omitted.

### V. Quadratic In the Denominator

24.  $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C, \quad a \neq 0$
25.  $\int \frac{bx + c}{x^2 + a^2} dx = \frac{b}{2} \ln|x^2 + a^2| + \frac{c}{a} \arctan \frac{x}{a} + C, \quad a \neq 0$
26.  $\int \frac{1}{(x-a)(x-b)} dx = \frac{1}{a-b} (\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$
27.  $\int \frac{cx + d}{(x-a)(x-b)} dx = \frac{1}{a-b} [(ac+d) \ln|x-a| - (bc+d) \ln|x-b|] + C, \quad a \neq b$

### VI. Integrands Involving $\sqrt{a^2 + x^2}$ , $\sqrt{a^2 - x^2}$ , $\sqrt{x^2 - a^2}$ , $a > 0$

28.  $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + C$
29.  $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}| + C$
30.  $\int \sqrt{a^2 \pm x^2} dx = \frac{1}{2} \left( x \sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} dx \right) + C$
31.  $\int \sqrt{x^2 - a^2} dx = \frac{1}{2} \left( x \sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} dx \right) + C$

$$\begin{aligned}
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \\
\sin(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \\
\cos(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \\
(1+x)^p &= 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \dots \quad \text{for } -1 < x < 1 \\
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \quad \text{for } -1 < x < 1 \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \quad \text{for } -1 < x < 1
\end{aligned}$$