Common moment generating functions

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Here are the moment generating functions (MGFs) for some of the distributions we have discussed. Unless otherwise specified, the MGF is defined for all real t.

1) Bernoulli $Ber(p)$:

$$M(t) = 1 + p(e^t - 1) \tag{1}$$

This is easy to derive directly from the definition.

2) Binomial $Bin(n, p)$:

$$M(t) = \big(1 + p(e^t - 1)\big)^n \tag{2}$$

This is derived using the binomial theorem, as shown in class. Note this is also the nth power of the MGF for $Ber(p)$; this is not an accident, as we will discuss later in the semester.

3) Poisson distribution $Poisson(\lambda)$:

$$M(t) = e^{\lambda(e^t - 1)} \tag{3}$$

This uses the Taylor series for e^x.

4) Exponential distribution $Exp(\lambda)$:

$$M(t) = \frac{\lambda}{\lambda - t}, \quad t < \lambda. \tag{4}$$

This is easily computed from the definition.

5) Standard normal distribution $N(0, 1)$:

$$M(t) = e^{t^2/2} \tag{5}$$

This is most easily computed by a trick involving completing squares. See the text.