Math 322 Sect. 2
Homework #8

Due Wed. 3/11/15

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Please write this up and turn it in. Recall we defined the complex exponential function e^{x+iy} (x, y real) in terms of a power series

$$e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \ . \tag{1}$$

From this definition alone (without using any other properties of the complex exponential) one can show that for any complex number z and real t,

$$\frac{d}{dt} e^{zt} = z e^{zt} \ . \tag{2}$$

The purpose of this problem is to show you how one can deduce the other properties of the complex exponential *from this property alone*. For example, the formula

$$e^{it} = \cos(t) + i \sin(t) \ , \tag{3}$$

can be derived from Eq. (2) as follows.

(a) Consider the differential equation

$$y''(t) + y(t) = 0 \ , \quad y(0) = 1, \ \ y'(0) = 0 \ . \tag{4}$$

Find the solution in terms of $\sin$ and $\cos$.

(b) Now find a solution in the form $c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$ where λ_1 and λ_2 are roots of $\lambda^2 + 1 = 0$ and c_1 and c_2 are (possibly complex) coefficients determined by the initial conditions.

(c) Show that the solutions you found in (a) and (b) represent the same solution.

(This gives you *half* of Eq. (3); the other half can be derived by using the initial conditions $y(0) = 0$ and $y'(0) = 1$.)