mh

January 8, 2019

For more information on this example, see "The Markov chain Monte Carlo revolution" by Persi Diaconis, Bull. AMS 46 (2009). I'm using an English translation of Tolstoy's "War and Peace" as the source of my word frequency data, and the US Constitution as my test example.

```
In [1]: using PyPlot
```

```
In [2]: C=include("Cipher.jl")
```

```
Out[2]: Main.Cipher
```

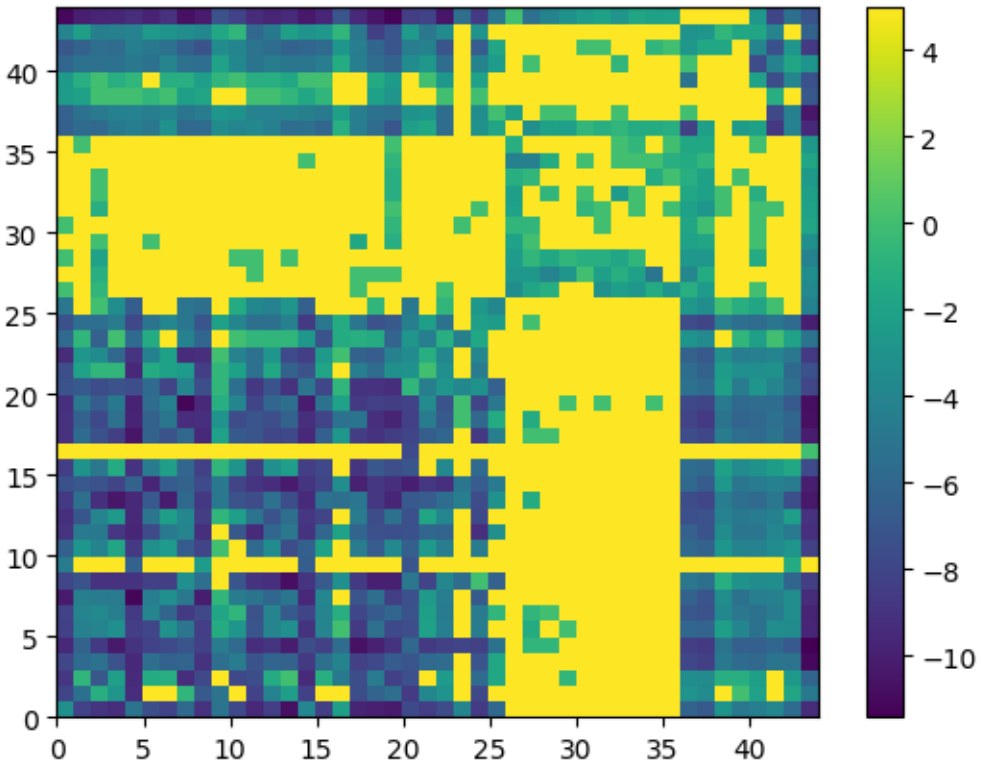
We compute a frequency matrix M_{ij} of i-to-j transitions for all letters of the alphabet i and j, using a big source text. We then define the "pairwise energy" $\phi_{ij} = -\log M_{ij}$.

```
In [3]: src=C.readtext("/tmp/pg2600.txt");
```

```
In [4]: pot=C.PairPotential(src)
```

```
Out[4]: (::getfield(Main.Cipher, Symbol("#pot#6")){Float64,Array{Float64,2},Dict{Char,Int64}}) (
```

```
In [5]: pcolor(pot())
        colorbar()
```




```
Out[5]: PyObject <matplotlib.colorbar.Colorbar object at 0x12b9723c8>

In [6]: pot('a','a')

Out[6]: -3.258096538021482
```

The "plaintext" is a copy of the US Constitution.

```
In [7]: ptext=C.readtext("/Users/kkylin/amalthea/constitution.txt");

In [8]: ptext[1:1000]

Out[8]: "provided by usconstitution.net------------------------------note repealed text is not n
```

We now make a random substitution cipher and apply it to obtain a "ciphertext."

```
In [9]: rc = C.RandomCipher()

Out[9]: Dict{Char,Char} with 44 entries:
          'w' => '"'
          '7' => 'p'
          'o' => 'm'
          '5' => '9'
```

```
          'h' => 't'
          'i' => 's'
          'r' => 'o'
          'q' => '1'
          ';' => '!'
          'a' => ','
          'c' => 'e'
          'p' => 'a'
          '9' => 'g'
          'x' => 'u'
          'u' => 'y'
          'd' => '3'
          'e' => 'b'
          'j' => '2'
          's' => 'n'
          '4' => '5'
          ',' => 'd'
          'z' => 'r'
          '0' => 'z'
          '3' => 'v'
          'n' => 'f'
            =>
```

```
In [10]: ctext = C.encrypt(rc,ptext);
```

```
In [11]: ctext[1:1000]
```

```
Out[11]: "aomcs3b3jh;jynemfn-s-y-smf4fb-wwwwwwwwwwwwwwwwwwwwwwwwwwwwwfm-bjobab, b3j-bu-jsnjfm-j
```

We can recover the plaintext from the ciphertext.

```
In [12]: C.decrypt(rc,ctext)[1:1000]
```

```
Out[12]: "provided by usconstitution.net------------------------------note repealed text is not
```

All that decrypt() does is invert the cipher (as a permutation) and apply it.

```
In [13]: irc=C.invertcipher(rc)
```

```
Out[13]: Dict{Char,Char} with 44 entries:
           'w' => '-'
           '7' => '6'
           'o' => 'r'
           '5' => '4'
           'h' => 'b'
           'i' => 'k'
           'r' => 'z'
           'q' => '2'
           ';' => 'y'
```

```
'a' => 'p'
'c' => 'v'
'p' => '7'
'9' => '5'
'x' => '\''
'u' => 'x'
'd' => ','
'e' => 'c'
'j' => ' '
's' => 'i'
'4' => '.'
',' => 'a'
'z' => '0'
'0' => '!'
'3' => 'd'
'n' => 's'
  =>
```

```
In [14]: C.encrypt(irc,ctext)[1:1000]
```

```
Out[14]: "provided by usconstitution.net------------------------------note repealed text is not
```

Based on this, for a string s and a cipher c we define the energy

$$E(c,s) = \sum_i \phi_{c(s_i),c(s_{i+1})}.$$

Note

$$e^{-E(c,s)} = \prod_i M_{c(s_i),c(s_{i+1})}.$$

So lower energy means the text is closer to having the correct statistics.

Here is the energy of the plaintext, computed two ways:

```
In [15]: e0=C.energy(pot,C.IdCipher(),ptext)
```

```
Out[15]: -423749.3414496625
```

```
In [16]: C.energy(pot,irc,ctext)
```

```
Out[16]: -423749.3414496625
```

Let's run some Monte Carlo to try to decode this.

```
In [17]: @time cmin,emin,el=C.mh(pot,ctext,5000)
```

```
  8.149655 seconds (313.50 k allocations: 27.242 MiB, 0.10% gc time)
```

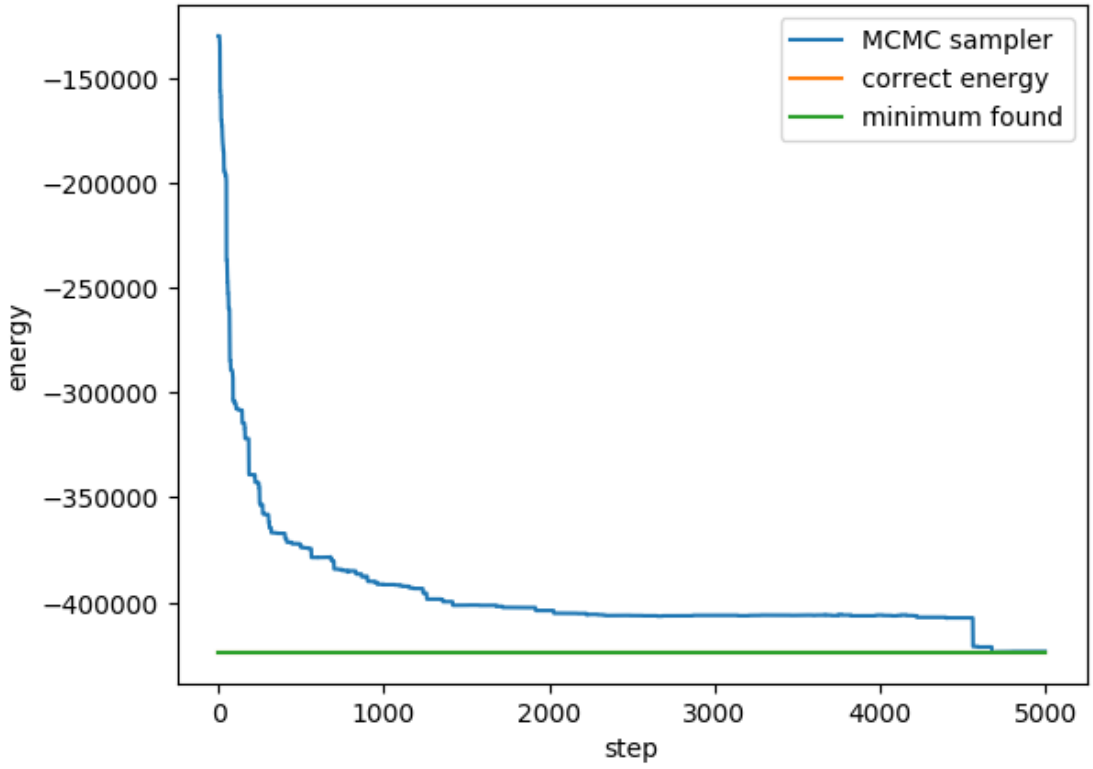
```
Out[17]: (Dict('w'=>'-','7'=>'2','o'=>'r','5'=>'8','h'=>'b','i'=>'k','r'=>'\'','q'=>'9',';'=>'y'
```

How good is the energy of the solution we found compared to the plaintext?

```
In [18]: emin/e0
```

```
Out[18]: 1.0003150226404838
```

```
In [19]: plot(el,label="MCMC sampler")
         plot([1,5000],[e0,e0],label="correct energy")
         plot([1,5000],[emin,emin],label="minimum found")
         legend()
         xlabel("step")
         ylabel("energy")
```




```
Out[19]: PyObject <matplotlib.text.Text object at 0x12eeb1fd0>
```

Does it really work? Let's try decrypting the ciphertext with it.

```
In [20]: dctext = C.encrypt(cmin,ctext);
```

```
In [21]: dctext[1:1000]
```

```
Out[21]: "provided by usconstitution.net------------------------------note repealed text is not
```

Let's count the percentage of characters that were correctly decoded.

```
In [22]: sum(map(==,dctext,ptext))/length(ptext)
```

```
Out[22]: 0.9958067952582788
```

The Monte Carlo is randomly initialized, so if we run it again we will get a different answer.

```
In [23]: @time cmin,emin,el=C.mh(pot,ctext,5000)
```

```
  8.326295 seconds (58.72 k allocations: 14.062 MiB, 0.09% gc time)
```

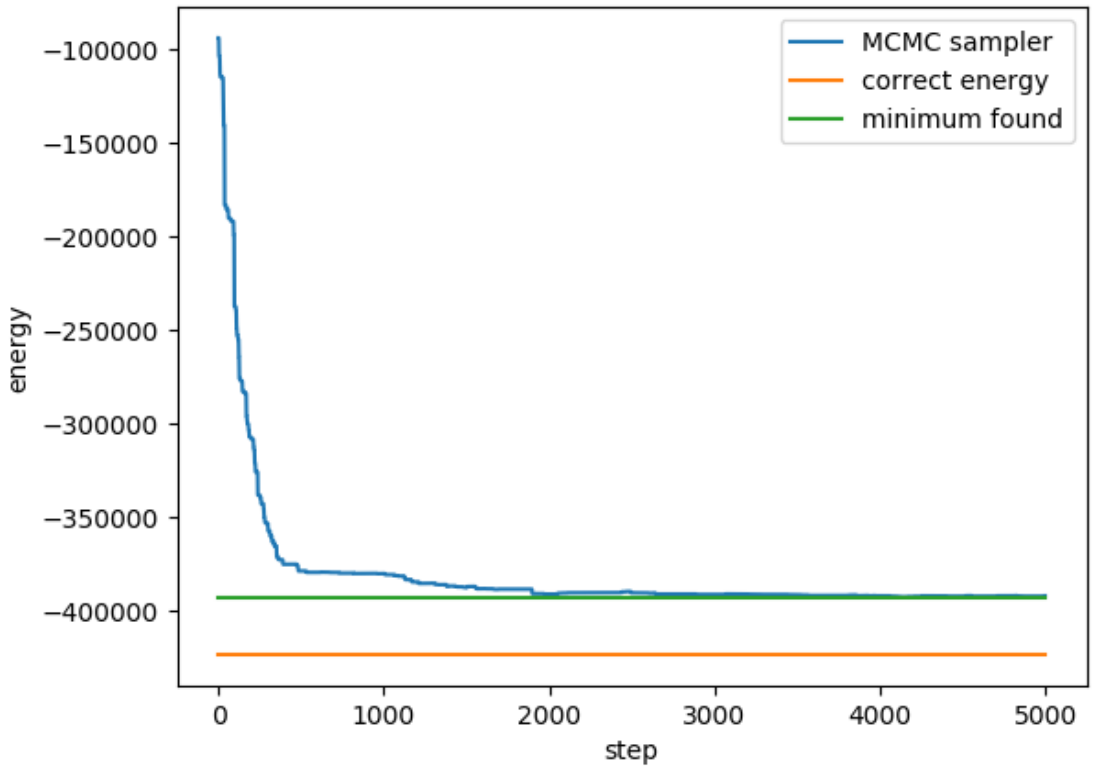
```
Out[23]: (Dict('w'=>'-','7'=>'7','o'=>'t','5'=>'0','h'=>'.','i'=>'\'','r'=>'j','q'=>'!',';'=>'b'
```

Hm, things look different this time: we don't get as close to the plaintext energy.

```
In [24]: emin/e0
```

```
Out[24]: 0.9273957326445056
```

```
In [25]: plot(el,label="MCMC sampler")
         plot([1,5000],[e0,e0],label="correct energy")
         plot([1,5000],[emin,emin],label="minimum found")
         legend()
         xlabel("step")
         ylabel("energy")
```



```
Out[25]: PyObject <matplotlib.text.Text object at 0x12eeacef0>
```

Does it work? Let's try.

```
In [26]: dctext = C.encrypt(cmin,ctext);

In [27]: dctext[1:1000]

Out[27]: "ptrvodide.beusfrns o u ornkni ------------------------------nr ietipialide ix eosenr e
```