Expectation of sums of random variables

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October 5, 2019

Here is a better explanation of the binomial example from Friday. Consider a sequence of n trials, each with probability of success p. The sample space Ω consists of the 2^n possible outcomes, each represented by a sequence of 0's and 1's: $\Omega = \{(\omega_1, \cdots, \omega_n) | \omega_i \in \{0,1\}\}$. We can define a random variable X so that for each outcome $\omega = (\omega_1, \cdots, \omega_n) \in \Omega$, $X(\omega)$ is just the number whose binary expansion is $\omega_1 \cdots \omega_n$. For example, for $n = 3$, $X(011) = 3$. We can then define

$$g_i(x) = i\text{th binary digit of } x. \tag{1}$$

For example

$$\begin{aligned} g_1(0) &= 0 \\ g_1(1) &= 1 \\ g_1(2) &= 0 \\ g_1(3) &= 1 \\ &\vdots \end{aligned}$$

and

$$\begin{aligned} g_2(0) &= 0 \\ g_2(1) &= 0 \\ g_2(2) &= 1 \\ g_2(3) &= 1 \\ &\vdots \end{aligned}$$

etc.

With this representation, we then have $X_i = g_i(X)$, where X is the random variable above and $X_i = 1$ if the ith trial succeeds and $X_i = 0$ if it fails. That is, the X_i are Bernoulli random variables. The rest is as shown in class: from the formula

$$E[g(X)] = \sum_k g(k) \cdot P(X = k) \tag{2}$$

we have $E[X_1 + \cdots + X_n] = E[g_1(X) + \cdots + g_n(X)] = E[g_1(X)] + \cdots + E[g_n(X)] = np$.