Homework 8

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Due Tuesday 3/31 at 11:59p on Gradescope

Graded problems.

- Exercise 1.74 from the text(*)
- Exercise 1.77 from the text

A1 Consider a Markov chain X_n with state space $S = \{0, 1, 2, \cdots\}$ and transition probabilities

$$p(x, x+1) = p_x \tag{1a}$$
$$p(x, x-1) = q_x \qquad \text{if } x > 0 \tag{1b}$$
$$p(0, 0) = q_0 \qquad \text{if } x > 0 \tag{1c}$$

(All other transitions have zero probability.) In the above, $p_0, p_1, \cdots$ are numbers in $(0, 1)$, and $q_x = 1 - p_x$. This generalizes the reflected random walk example.

(a) Suppose the chain has a stationary distribution π. What equation does it satisfy?

(b) Using ~~$\sum_x \pi(x) = 1$~~ the fact that $\lim_{x\to\infty} \pi(x) = 0$ (because $\sum_{x=0}^{\infty} \pi(x) = 1$), show that *if* there is a stationary distribution, then it must satsify detailed balance.

(c) In terms of the ratios p_x/q_{x+1}, give a condition that guarantees the existence of a stationary distribution.

A2 For the reflected random walk with $p = q = \frac{1}{2}$, let $N > 0$ be fixed, and consider

$$g(x) = E_x V_{0,N} \tag{2}$$

where

$$V_{0,N} = \min\{n \geqslant 0 \mid X_n = 0 \text{ or } X_n = N\}. \tag{3}$$

Show that $g(x) = Nx - x^2$ for $0 < x < N$ (**).

A3 Show that if a Markov chain with transition probability function $p(x, y)$ is irreducible and has a stationary distribution $\pi(x)$, then $\pi(x) > 0$ for all states x (***).

Suggested problems. Exercises 1.75, 1.76 from the text.

Hints. Here are some hints for those who want them.