Lecture 1 notes

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Examples:

- Queue
- Internet
- PageRank
- Geiger counter
- Brownian motion
- A simulation of Gambler's Ruin

Definitions:

- *Law of alternatives.* Suppose $B_1, \cdots$ are a collection of events such that (i) $B_i \cap B_j = \emptyset$ for $i \neq j$, (ii) $\cup_i B_i = \Omega$, and (iii) $P(B_i) > 0$ for all i. Then for any event A we have

$$P(A) = \sum_i P(A|B_i)P(B_i). \tag{1}$$

The sum can be finite or infinite.

- *Law of successive conditioning.* Suppose $A_1, \cdots, A_n$ are a collection of events with positive probability. Then

$$P(A_1 \cap \cdots \cap A_n) = P(A_1|A_2, A_3, \cdots, A_n) \cdot P(A_2|A_3, A_4 \cdots, A_n) \cdot \cdots \cdot P(A_{n-1}|A_n) \cdot P(A_n). \tag{2}$$

Note: the terminology I use is not universally used, though the results are very much standard.