Lecture 7 notes

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Today we finished proving the results in Sect. 1.3, and began discussing stationary distributions (Sect. 1.4).

We looked at two main examples, random walk around a circle and Gambler's Ruin. The latter shows that stationary distributions need not be unique, and that when there is more than one, there must be infinitely many. Indeed, we essentially proved the following result:

Theorem. if $\pi_1, \cdots, \pi_k$ are stationary distributions and $c_1, \cdots, c_k$ are real numbers in $[0, 1]$ such that $\sum_i c_i = 1$, then

$$c_1\pi_1 + \cdots + c_k\pi_k \tag{1}$$

is again a stationray distribution.

A linear combination where the coefficients lie in $[0, 1]$ and sum to 1 (as in Eq. (1)) is called a *convex combination.*

The result above can be shown directly from the definition of stationary distributions, or by the equivalnet linear algebra interpretation, i.e., $\pi \cdot P = \pi$. I'll say more about the connection between stationary distributions and linear algebra later.