## Lecture 11 notes

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Today we discussed

- Theorems 1.20, 1.21, and 1.22
- The detailed balance condition.

Notes:

1) The text gives a proof of Theorem 1.22 . One can also understand the relationship between 1.21 and 1.22 in the following way. First, consider the equation

$$
\begin{equation*}
\sum_{k=1}^{n} f\left(X_{k}\right)=\sum_{y \in S} f(y) N_{n}(y) . \tag{1}
\end{equation*}
$$

Divide both sides by $n$, let $n \rightarrow \infty$, commute the limit and the sum, then apply Theorem 1.21 yields 1.22 . Conversely, assuming 1.22 , one can obtain 1.21 by letting, for a fixed state $y$,

$$
f(x)= \begin{cases}1, & x=y  \tag{2}\\ 0, & x \neq y\end{cases}
$$

2) Theorem 1.22 can be used to obtain the average daily operational cost of maintaining the machinery in Example 1.38, even in situations where the "cost" is negative (i.e., one gets paid to replace some of the parts).
