None of these problems requires a calculator for doing the integration. In all cases, in order to receive full credit, you should show enough work that it is clear that you can do the problem correctly and completely without using a calculator [emphasis added]. Of course, there is nothing wrong with using calculator as permitted by course policy to CHECK an answer.

1. For the following integral, choose an appropriate trigonometric substitution, and make the substitution completely in the integral. After completing the substitution (so that there is no variable $x$ in the integral), IT IS NOT NECESSARY TO SIMPLIFY OR TO FIND THE INTEGRAL.

$$\int \frac{x^2}{\sqrt{14 + x^2}} \, dx.$$  

SOLUTION. For typing convenience, let $\sigma = \sqrt{14}$. Choose $x = \sigma \tan(u)$. Then $dx = \sigma \sec^2(u) \, du$ and $14 + x^2 = 14 + 14 \tan^2(u)$. Thus the integral is

$$\int \frac{x^2}{\sqrt{14 + x^2}} \, dx = \int \frac{14 \tan^2 u}{\sqrt{14 + 14 \tan^2 u}} \sigma \sec^2 u \, du.$$  

STOP! The instructions say

After completing the substitution (so that there is no variable $x$ in the integral), IT IS NOT NECESSARY TO SIMPLIFY OR TO FIND THE INTEGRAL.

2. Show how to expand in partial fractions (i.e., how to split into partial fractions). You don’t have to do any calculations. Just show what a partial fraction expansion would look like.

$$\frac{x}{(x^2 + 16)(x - 4)^2}.$$  

SOLUTION. 

$$\frac{x}{(x^2 + 16)(x - 4)^2} = \frac{Ax + B}{x^2 + 16} + \frac{C}{x - 4} + \frac{D}{(x - 4)^2} \quad \text{for constants } A, B, C, D.$$  

STOP! The instructions say

You don’t have to do any calculations.

Just show what a partial fraction expansion would look like.

3. Split (decompose) into partial fractions, being as efficient as possible in finding the constant numerators in the partial fractions: $\frac{1}{x^2 - 16}$

SOLUTION. 

$$\frac{1}{x^2 - 16} = \frac{1}{(x - 4)(x + 4)} = \frac{A}{x - 4} + \frac{B}{x + 4}.$$  

Efficient way to find $A$ and $B$, from the last equality:

For $A$: Multiply by $x - 4$ (this “cancels” the $x - 4$ in the denominators); set $x = 4$ (this gets rid of the $B$ term). We get $1/(4+4) = A$, so $A = 1/8$.

Similarly for $B$: Multiply by $x + 4$, set $x = -4$. We get $1/(-4-4) = B$, so $B = -1/8$.

So the partial fraction expansion is

$$\frac{1}{x^2 - 16} = \frac{1/8}{x - 4} + \frac{-1/8}{x + 4}.$$