None of these problems requires a calculator for doing the integration. In all cases, you should show enough work that it is clear that you can do the problem correctly and completely without using a calculator [emphasis added]. Of course, there is nothing wrong with using calculator as permitted by course policy to CHECK an answer.

3. Split (decompose) into partial fractions, being as efficient as possible in finding the constant numerators in the partial fractions: \[
\frac{1}{x^2 - 16}
\]

**SOLUTION.** (Details posted elsewhere.)

\[
\frac{1}{x^2 - 16} = \frac{1/8}{x - 4} + \frac{-1/8}{x + 4}.
\]

**COMMENT.** The equation just given is the “final” answer to the problem, “split into partial fractions”. This shows the partial fractions. The final answer is NOT \( A = \text{something}, B = \text{something} \).

4. Evaluate the following integral, without using tables. \[
\int \frac{x^2}{x^2 - 16} dx
\]

**IMPORTANT COMMENT.** The integrand is a rational function. As discussed in class frequently, and in the textbook, the first things you should check are:

- Can the denominator be factored (in this case, the answer is obviously YES);
- If so, factor and think about using partial fractions, and then you should start with the following question:
- Is \( \text{deg(numerator)} < \text{deg(denominator)} \); if the answer is NO, as it obviously is here, then you do some kind of algebraic manipulation to get a rational function in which \( \text{deg(num)} < \text{deg(denom)} \).

**SOLUTION.** (The immediately preceding problem, Problem 3, was all about partial fractions and involved a function VERY similar to the function given here. Therefore, in addition to the important comment just made, the first thing you should think about is partial fractions.)

\[
\frac{x^2}{x^2 - 16} = \frac{x^2 - 16 + 16}{x^2 - 16} = 1 + \frac{16}{x^2 - 16}
\]

\((A \text{ problem exactly like this was done in class the week of the exam.})

Now use Problem 3:

\[
1 + 16 \left( \frac{1/8}{x - 4} + \frac{-1/8}{x + 4} \right) = 1 + \frac{2}{x - 4} - \frac{2}{x + 4}.
\]

This is very easy to integrate:

\[
\int \frac{x^2}{x^2 - 16} dx = x + 2 \ln|x-4| - 2 \ln|x+4| + C.
\]

**ANOTHER COMMENT.** This is NOT a trig substitution problem. As discussed in the textbook and in class, one might use a tan substitution when the integrand contains \((a^2 + x^2)\), and a sin substituion when the integrand contains \(\sqrt{a^2 - x^2}\).