1. Evaluate the integral \( \int_{0}^{1} \frac{x}{1+x} \, dx \).

As usual, you should give an exact answer (unless you are specifically asked for a decimal approximation). This is especially important when the instructions for the assignment specifically say “closed calculator”.

Comments.

Serious high school algebra errors:

\( \frac{a}{b+c} \) is not equal to \( \frac{a}{b} + \frac{a}{c} \).

\( \frac{x}{1+x} \) is not equal to \( \frac{x}{1} + \frac{x}{x} = x + 1 \).

Similar: \( x(1+x)^{-1} \) is not equal to \( x + 1 \).

[Take \( x = 5 \). Does \( \frac{5}{1+5} = 5 + 1 \)??].

\( \frac{a}{b+c} \) is not equal to \( a + \frac{1}{b+c} \).

\( \frac{x}{1+x} \) is not equal to \( x + \frac{1}{1+x} \).

Serious first semester calculus errors:

\( \int f(x)g(x) \, dx \) is not equal to \( \int f(x) \, dx \int g(x) \, dx \)

\( \int \frac{x}{1+x} \, dx \) is not equal to \( \int x \, dx \int 1/(1+x) \, dx \)

\( \int f(x)g(x) \, dx \) is not equal to \( f(x) \int g(x) \, dx \)

\( \int \frac{x}{1+x} \, dx \) is not equal to \( x \int 1/(1+x) \, dx \)

Solution (brief): \( 1 - \ln(2) \).

(Can EASILY be done by simple, correct high-school-algebra manipulations, OR by substitution.)