PART II.

Now let’s go “backwards”, start with a function $f(x)$ (no power series yet), and use the derivatives to construct a power series using $f(x)$:

We use the formulas above,

$C_0 = f(0)$.

$C_1 = \frac{1}{1!} f'(0) = f'(0)$.

$C_2 = \frac{1}{2!} f''(0) = \frac{1}{2} f''(0)$.

$C_3 = \frac{1}{3!} f'''(0)$.

And in general

$C_n = \frac{1}{n!} f^{(n)}(0)$.

Use these coefficients to construct a power series about $x = 0$:

$C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots$

$= f(0) + f'(0) x + \frac{1}{2} f''(0) x^2 + \frac{1}{3!} f'''(0) x^3 + \ldots + \frac{1}{n!} f^{(n)}(0) x^n + \ldots$

This series is called the Taylor series of $f(x)$ about $x = 0$. (Sometimes slightly different words are used.)

By a “Miracle Of Mathematics” it turns out that this power series is equal to the function $f(x)$ that we started with in some interval of convergence. (This statement is true for all functions we meet in Math 129; it is possible to find some “weird” functions for which this is not true.)

So,

$$f(x) = f(0) + f'(0) x + \frac{1}{2} f''(0) x^2 + \frac{1}{3!} f'''(0) x^3 + \ldots + \frac{1}{n!} f^{(n)}(0) x^n + \ldots.$$